Lecture 5: Euler’s formula & “applications”

**Aim Lecture** Multn & computing powers/roots best

**Lemma** \((\cos \theta + i \sin \theta)(\cos

**Proof** LHS =

**remark** Here, mult numbers corresponds to
Geom Interpretation

\[ z = \cos \phi + i \sin \phi, \ w = r( \]

zw “is” w

Euler’s Formula For \( \theta \in \)

More gen,

\[ e^{a+bi} := \]

e.g. \( e^{i \pi /2} \)
Q Why is this defn sensible?

A 1. It extends defn for reals & in fact you’ll learn in 2nd yr complex analysis courses that it’s

2. We have the desirable

\textbf{Formula} \ e^{z+w} =

For \ z =

\textbf{Proof} \ LHS =

RHS =
3. In MATH1241 you’ll see following formulae

\[ e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \ldots \]

\[ \sin x = \]

\[ \cos x = \]

Suggests

\[ e^{ix} = 1 + ix + i^2 \frac{x^2}{2!} + i^3 \frac{x^3}{3!} + i^4 \frac{x^4}{4!} \ldots \]

**Formulae** For \( n \in \mathbb{Z} \)

1. \( e^{i(\theta+2n\pi)} = \)

2. \( (e^{i\theta})^n = \)

**Proof**

1. holds as \( \cos, \sin \)

2. Case a), \( n \geq 1 \)

\( (e^{i\theta})^n = \)
Case b), $n = 0$

Case c), $n < 0$
\[ e^{in\theta} e^{-in\theta} = \]
so $e^{in\theta} = \]

An immediate corollary is

De Moivre’s Thm

Proof
\[ e^{i\pi/2} = i, i^2 = -1 \implies \]

Powers of complex numbers
Q Find \((\sqrt{3} - i)^{100}\)

Dumb method

Good Method: Write \(z = \sqrt{3} - i\) in

\(|z| = \)

\(\text{Arg} \, z = \)

\(z = \)

\(z^{100} = \)
**Formulae** For $z, w \in$

1. $|\frac{z}{w}| =$

2. $\text{Arg } zw =$

3. $|z^n| =$

4. $\text{Arg } z^n$

**Proof** half of 1 & 2 only. Let $z = re^{i\theta}, w =$

$\frac{z}{w} =$

Hence, $|\frac{z}{w}| =$

$\text{Arg}$

e.g. Let $z = -1 + i, w = 1 + \sqrt{3}i$. Find
argument and modulus of $zw$.

**e.g.** What’s $|\bar{z}|$?