Q In what sense is \( \mathbb{C} \) with addn

**Aim Lecture** Answer this question by

**Fields**: Let \( \mathbb{F} \) be a set.

E.g.

**Imprecise Defn** \( \mathbb{F} \) is a field (technical name for system of numbers) if

More precisely, suppose

i) an addn rule on \( \mathbb{F} \) assigning
ii) a multn rule

Say $\mathbb{F}$ (with these 2 rules) is a field if it satisfies

For any $x, y, z \in \mathbb{F}$

1. Associative Law of Addition:

2. Commutative Law of Addition:

3. Existence of Zero:

4. Existence of Negatives:
5. Associative Law of Multn:

6. Commutative Law of Multn:

7. Existence of One:

8. Existence of Inverses: If $x \neq$

9. Distributive Law:

**E.g.** $F = \mathbb{Q}$,

**Defn** Laws 1-9 are called **Subtrn/Division** [H] Using 4. we can de-
fine subtraction as adding the negative. Sim division is

Subtle Point [H]: Need uniqueness of negatives and inverses to

**Thm** [H]: 1. \( \mathbb{F} = \mathbb{C} \) is

2. Subtrn and division in \( \mathbb{C} \) is given by adding the negative and multiplying

**Proof** 1. Need to check 9 axioms. We’ll only check axiom 2.

2. We’ll check subtrn only.
0 = 0 + 0i is the zero since

The negative of \( c + di \) is \(- (c + di) = \)

Let’s add the negative of \( c + di \) to

This is our defn

**Manipulating complex numbers** To multiply, divide etc. complex numbers, we usually use

e.g. 1 \((3+i)(3-i)\)
2. \((1+i)^2 = \)

The 9 laws above yield lots of other formulae we are familiar with in the algebra of real numbers. E.g.

**Formula** For \(w, x, y, z \in \mathbb{C}\)

**Proof**[H] \(\left(\frac{w}{x}\right)\left(\frac{y}{z}\right) = \)

Need \(x^{-1}z^{-1} = \)

We check this

We usually use this formula to divide
e.g. 3

We have more

**Formulae** For $w, z \in$

1. $\bar{z}$

2. $Re \ z =$

3. $z - w$

4. $\bar{zw}$

5. $z$ is real iff

Some **Proofs**
e.g. 4 Show that $\bar{z}w + zw\bar{w}$ is real.

e.g. 5 Solve the following for $z$.

$z^2 + iz = yz$.  
