Lecture 2: Arithmetic of Complex Numbers

Aim of Lecture: Enlarge set of

Q What is
Possible A A real number

Motivation for complex numbers
Q Suppose graph of a polynomial fn is

e.g. $y = x^2 + bx + c$
What’s $d$?

A

In e.g. quadratic formula $\implies$

Problem: If

However, if we pretend the expressions exist & obey
Ex If $y = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots a_n$ has an axis of symmetry then it is

**Conclusion:** It is often useful in maths to pretend expressions such as

Later will see more serious applications of this idea. The above expressions don’t represent amounts but are artificial

Today’s **Q** What expressions should we treat as numbers and how

**N.B.** If usual laws of
Defn Let $i$

A complex number is an

Let $\mathbb{C}$ denote

e.g.

N.B. $\mathbb{C}$ contains the set of

Q When are 2

A Assuming usual

$a + bi = c + di$ \implies
since

Suggests

**Defn** \( a + bi = c + di \) means

**Defn** Let \( z = a + bi, a, b \in \mathbb{R} \) be a complex number. The real

Complex numbers are equal iff

The complex conjugate

e.g.

Q What are sensible ways to add,
For complex numbers $z =$

**Addn** Define

e.g.

**Subtrn** Define

e.g.

**Multn** A priori not even clear

Let’s see what happens if
Suggests defn

**Neat Formula** $z \bar{z}$

**Proof**

**Division** Assume $w \neq 0$ i.e.

Need trick to suggest formula for

Suggests defn

**Note** $c^2$