Lecture 10: Applications. Laws of Arithmetic for Vectors

**Aim of Lecture**

a) We’ll formulate some practical problems using $n$-tuples.

b) Field axioms gives basic laws for simplifying manipulating numbers.

Let $\mathbb{F} = \text{field e.g.}$

**Defn** Given $\mathbf{v}, \mathbf{w} \in \mathbb{F}^n$, define the negative of

Define vector subtraction

**Relative Velocity** Suppose an observer
moving

sees an object

Define relative velocity (of object wrt

\[=\text{velocity object appears to be moving at}\]

\textbf{e.g. 1} Coyote moves 40 kmh}^{-1} \text{ NE pursuing road runner moving 20kmh}^{-1} \text{ at } 30^\circ \text{ east of N. What’s apparent speed of}

\text{Coords of} \quad \text{coords of}
Rel vel =

apparent speed =

**Line of Best Fit**

*e.g.* 2 Lex believes

i.e. if $x =$ amount of

\[ y = \text{strength of} \]

then $y = mx + b, m < 0$.

**Q** Formulate precise maths problem to determine

**A** For $i = 1, \ldots, n$ let

\[ x_i = \text{radiation} \]

\[ y_i = \]
Plot data points

For any given $m, b$ predicted value of $y_i$ is

\textit{i}-th error in prediction

Define error

\begin{align*}
\mathbf{x} &= \\
\varepsilon &= 
\end{align*}
Best choice of
Math formulation: Find $m, b$ which

Will solve in later courses

**Laws of Arithmetic for $\mathbb{F}^n$:** $\mathbb{F} =$

For vectors $u, v, w \in \mathbb{F}^n$ and scalars

1. Associative Law of Addition:

2. Commutative Law of Addition:

3. Existence of Zero:

4. Existence of Negatives:
6. Associative Law for Scalar Multn:

7. $1 \mathbf{v} =$

9. Scalar Distributive:

10. Vector Distributive:

Cor The above “axioms” hold for

Why?

Some proofs of thm: Commutativity of
addn seen for geom vectors. In \( \mathbb{F}^2 \)

Let’s check 1 other e.g. vector distributive.

In \( \mathbb{F}^2 \), \( \lambda((v_1, v_2) + \)

LHS =

Can also be “seen” geom using

Like field axioms, laws let you simplify vec-
tor expressions.

e.g. $3 (4 \mathbf{i} + 5 \mathbf{j}) - 2 (\mathbf{i} + \mathbf{j})$

From the basic laws also follow lots of other useful formulae

**Propn** For $\mathbf{u}, \mathbf{v}, \mathbf{w}$

a) (cancellation)

b) $0 \mathbf{v} =$

a) & b) also hold for geometric vectors

**Proof** a)
b)