

# MATH5975

## INTRODUCTION TO STOCHASTIC ANALYSIS

Modern theory of security markets rely on advanced mathematical and statistical methods that are used to model, forecast and manage risk in complex financial transactions. Beginning from the groundbreaking work of Black and Scholes (1973) on the arbitrage pricing of European call options, it became clear that Stochastic Analysis is an indispensable tool for the theory of financial markets, derivation of prices of standard and exotic options and other derivative securities, hedging related financial risk, as well as managing the interest rate risk. In this course, you will learn the basic concepts of Stochastic Analysis, such as: the Brownian Motion, the stochastic integral, Itô's formula, stochastic differential equations, equivalent change of a probability measure, integral representation of martingales with respect to a Brownian filtration, relations to second order PDEs, and the Feynman-Kac formula. Some concepts will be illustrated by examples relevant for financial applications. This course should be seen as an indispensable prerequisite for the course MATH5816 Continuous Time Financial Modelling.

*Outline of the course:*

1. Conditional expectations, filtrations, martingales, stopping times.
2. Continuous local martingales, localisation, the Doob-Meyer decomposition.
3. Standard Brownian Motion: construction, basic properties, sample path properties.
4. The Itô stochastic integral, Itô's formula, continuous semimartingales.
5. Stochastic differential equations.
6. Lévy's characterisation theorem.
7. Predictable representation theorem.
8. Feynman-Kac formula.
9. Girsanov's theorem.

*Recommended textbooks:*

Robert J. Elliott and P.E. Kopp: *Mathematics of Financial Markets*. Springer, Berlin Heidelberg New York, 1999.

Thomas Mikosch: *Elementary Stochastic Calculus with Finance in View*. World Scientific, Singapore, 1999.

Damien Lamberton and Bernard Lapeyre: *Introduction to Stochastic Calculus Applied to Finance*. Chapman and Hall, London, 1996.

Ioannis Karatzas and Stephen Shreve: *Brownian Motion and Stochastic Calculus*. Springer, Berlin Heidelberg New York, 1988.