MATH5816
CONTINUOUS TIME FINANCIAL MODELLING

The course focuses on continuous-time modelling of financial market under deterministic interest rates. The main goal of the course is a detailed study of the classical Black-Scholes model and its variants. We introduce the concept of a continuously rebalanced portfolio, and we examine the arbitrage-free property of the model by examining the existence and uniqueness of a martingale probability measure (using Girsanov's theorem). We provide two alternative proofs of the Black-Scholes option pricing formula. The first relies on the calculation of the replicating strategy; it thus requires solving the Black-Scholes PDE. The second method is based on probabilistic considerations and it makes direct use of the risk-neutral valuation formula. The second part of the course is devoted to an analysis of the concept of volatility. First, we introduce and study the notions of historical and implied volatilities. Subsequently, we present the approach known as the implied local volatility modelling. In this approach, the observed market prices at a given date (and thus the observed smiles and skews) are taken as inputs. We show the existence of a diffusion-type model which, by construction, is fitted to the observed term structure of volatility smile. In the next part of the course, we study contingent claims of American style in the Black-Scholes set-up. We explain that the valuation of an American claim is closely related to a specific optimal stopping problem. We show that for the purpose of arbitrage valuation, the maximization of the expected discounted payoff should be done under the martingale measure. The value of an American put option is compared to the value of a corresponding European put. The final section of the course is devoted to cross-currency derivatives.

Outline of the course:

1. Black-Scholes model and its variants: self-financing strategies, arbitrage opportunities, martingale measures, option pricing formula, replication of European contingent claims, Black-Scholes PDE, completeness, Black model, currency options.

3. Volatilities: historical volatility, implied volatility, volatility surface, risk-neutral marginal distributions, local volatility models.

2. American claims: American call and put option, rational exercise time, early exercise premium representation, optimal exercise boundary.

3. Cross-currency derivatives: domestic and foreign martingale measures, currency forward contracts and options, options on a foreign stock.

Recommended textbooks:

Marek Musiela and Marek Rutkowski: *Martingale Methods in Financial Modelling.* 2nd ed. Springer, 2005.

Robert J. Elliott and Peter E. Kopp: *Mathematics of Financial Markets.* Springer, 1999.

Rose-Anne Dana and Monique Jeanblanc: *Financial Markets in Continuous Time.* Springer, 2003.

Steven Shreve: *Stochastic Calculus for Finance II. Continuous Time Models.* Springer, 2004.