

Evolution versus Intelligent Design: a mathematician's view

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Introduction

This paper gives a very personal look at how mathematics might augment science in understanding the cosmic framework, and contribute to the debate between biologists and religious fundamentalists over evolution and intelligent design. Mathematics and science have quite distinct histories and methodologies, even though they mutually reinforce each other. I will argue that mathematics provides us with good reasons for being open-minded about the possibility that the world incorporates a ‘cosmic intelligence’—without being too precise about what this might be and what form it might take (or have taken). And I hope along the way to show you some really interesting but elementary mathematics related to a beautiful discovery by Lester Ford 70 years ago about circles associated to rational numbers. Oh, and I also want to cut down free will.

The increasingly strident debate on evolution versus intelligent design (ID) has caused both sides to overstate their positions. As a professional mathematician without religious connections, but with great respect for Darwin's crucially important ideas, I am naturally sympathetic to the scientific view. I was an atheist during most of my younger years, but a long study of mathematics has slowly led me towards a mild form of ID ; one that does not insist on a biblical rewriting of science, but which wants to acknowledge the remarkable coincidences, startling beauty and overpowering rigidity, structure and *rightness* in the mathematics underlying our world. This is an aspect of existence that is sadly unknown to a good part of the population, biological scientists perhaps included, so it deserves some press. Perhaps it may encourage others, who might be inclined to atheism, to be more open minded.

Many intelligent people are annoyed by the assuredness of religious fundamentalists, and react against the faith-based proclamations of organized religion by privately dismissing the whole business, although they may rarely come out and say so openly. These people want belief to be supported by *reasons*, and they fail to see much evidence in the world around them for any kind of clear organizing structure or grand design. Could it be that this is because they have *not learnt enough mathematics?*

Discussions of evolution versus ID usually call on arguments from biology, cosmology, physics or history, but are weak on mathematics. In this paper I will summarize the basic evolutionary picture, admit to a few of its problems, and introduce you to some simple yet beautiful arithmetic and geometry. Even if you have little knowledge, or perhaps interest in mathematics, I hope you will agree that the existence of such mathematical patterns might be a counterweight to the otherwise seemingly random nature of the world in which we live. And perhaps even a reason to be optimistic about the possibility of a Divine Creator.

Going more out on a limb, I will call on modern neuro-psychology and Einstein's theories of relativity to destabilize the familiar view of the universe as a dynamic place which is unfolding as we watch, along lines that we can potentially understand. I will suggest that *the world has already been created in its historical entirety*. We are just too low dimensional and internal to it to witness it across both space and time. This is consistent with the idea of an omniscient deity, whose existence probably implies that *everything is already known, and so any idea of free will must be an illusion*.

Evolution versus Intelligent Design

What is ID? In its strongest form, it is religious fundamentalism disguised as science, with its main aim to support a certain form of Christianity. It is opposed to the idea of evolution, which it sees as demeaning mankind, and hence by implication the Creator. This extreme form of ID refuses to acknowledge that the argument hinges around *evidence*, and so appears to be indifferent to the vast number of facts that scientists have uncovered that directly and indirectly support evolution.

On the other extreme, there is a much milder form of ID, which argues that the world in which we live was created by some form of 'intelligence' that is responsible for its design, and which might be called God, Allah, Jehovah, Krishna or something else, or might be left unnamed. From such a position evolution is not wrong, but is merely an aspect of the overall pattern. This milder form of ID has its natural opponents not *scientists*, but *atheists*.

The 'blind watchmaker' idea—vigorously promoted by Richard Dawkins and others—claims that there is no over-arching design or intelligence to the world, and that its history consists of purposeless bumbling along a path directed and constrained by biological principles like evolution, and physical laws like those of quantum mechanics and thermodynamics (entropy is increasing). This view is becoming increasingly disparaging of agnostic uncertainty. It seems to me also at odds with the splendor and richness of evidence from the mathematical world. The staunchly atheistic position expressed so well by Dawkins in his recent best-seller '*The God Delusion*' appears to be an overreaction to the unsubstantiated assertions and blind faith of fundamentalists, rather than a reasoned scientific position.

Yes, there is a lack of physical hard evidence for a God or Gods, but this might be weighed against the large amount of carefully observed mathematical

structure in our universe, which almost surely did not arise in a dynamical or evolutionary fashion, and points to the possibility of a larger unseen organizational aspect.

The image of a personal God who has created this world largely for our benefit is in many scientists' view rather unlikely. Unlikely but not impossible—for our seemingly puny aspect in the immensity of the cosmos is counterbalanced by the fact that we are so much more *interesting* than anything else in it—at least by current understanding. In another million years, if we have not managed to eradicate ourselves, we may be distributed much more widely in the galaxy or beyond, and may some day play a role even in the physical evolution of the universe. Those anti-religious arguments that rest on our diminutive scale may prove hollow if the universe is a stage set for mankind as a key player, in which the current age is just the beginning of Act 1. On the other hand, if many other intelligent species in the universe are one day discovered, the claim that God is primarily concerned with us largely collapses.

In any case, it is belittling the opposition when an atheistic scientist pretends that a bearded 'Father in the Sky' is the only manifestation of a supreme intelligence that needs to be argued against. It is tempting for some scientists to battle the opponents of evolution with all means at their disposal, but one's position may be weakened if one defends or attacks too forcefully, and Darwin's ideas are robust enough to not require our help. If the happy people of Kansas insist that their children be taught ID instead of, or side by side with, proper science, then civilization will not end. After some years, children taught this way may find that they are inadequately prepared for higher education, and they'll resent having been denied an opportunity to fully engage the modern world. In which case the program may be quietly shelved. Or not.

A mathematical perspective

A question/challenge for those who support the hypothesis of a universe without design: how many *interesting mathematical facts* do you know? If the answer is: *very few*, then perhaps your world view has a large slice missing—an aspect that might be even more relevant to the question of cosmic intelligence than the history of the solar system, the fossil record on planet earth, or the genetic sequence contained in our chromosomes.

The ancient Greeks, building on earlier Egyptian and Babylonian teachings, initiated logical mathematical reasoning more than 2500 years ago. Thousands of mathematicians working for three millennia have uncovered a vast and intricate, if largely secret, store of mathematical knowledge that compares favourably with the much more widely publicized achievements of those more recent upstarts: biology, chemistry and physics.

During this quest, mathematicians have often asked questions such as: How is the remarkable logical coherence of mathematics possible? Why does so much structure and beauty lie at the heart of a world which otherwise often seems chaotic and ugly? Why is mathematics so useful in understanding the physical

and chemical laws of the world?

Some physicists speculate that the laws of physics may have looked quite different in the very early stages of the universe. Mathematical structure on the other hand seems much more fixed. It appears indifferent to any evolutionary processes, and is the opposite of chaotic, random and arbitrary. One can perhaps liken it to a glorious cathedral in a largely formless desert, and the possibility that it was created or designed by something which might broadly be referred to as *intelligence* must not be discounted. This mathematical perspective does not advocate one religion over another, or even necessarily steer one towards religion.

Let's now summarize the rough argument for evolution, and acknowledge some difficulties.

The eons previous

The majority biologists' view is that the complexity of our world can be explained as a long and complicated evolutionary development, whose basic principles, and some of the details, were described by Darwin. Evolutionary theory has now been suitably augmented and expanded by both massive amounts of data (fossils, bones, genetic sequences and correlations, geological and meteorological information) as well as penetrating newer theories that show how many of our social behaviors and cultural constructs dance to Darwin's tune. Nevertheless, there are still unclear points, disagreements, and possible minor, or not so minor, variations to the theme that will probably be uncovered as the ongoing scientific work proceeds. It is unfair not to admit this point.

By current estimates the universe seems to be about 15 ± 5 billion years old, and our planet has been around for a few billion, although such numbers ought to be taken with a grain of salt. During the last billion years, physical and chemical conditions on earth arose which led to the formation of large molecules with replicating properties, which subsequently allowed them to become both plentiful and varied. At this point *natural selection* took charge—Darwin's simple but powerful principle that those organisms which are best adapted to a particular environment tend to reproduce more in it. A competition ensued, and molecules formed larger and larger conglomerates that had the ability to dismantle and absorb others, with eventually trillions of hungry microscopic creatures roaming the seas and competing for food. Over hundreds of millions of years this escalation led to a wide variety of plant and animal life forms in the seas, land, and air.

For the last three hundred million years or so, the dominant creatures were dinosaurs, who occupied most niches of the animal world and exhibited a wide range of adaptive strategies. Some sixty five millions years ago, the dinosaurs all dramatically died off, leaving the planet to small mammal-like creatures that subsequently evolved into larger mammals—earlier versions of horses, bears, sloths, whales, monkeys and so on. Five million years ago the monkeys had evolved into separate families including some more apelike creatures, and some-

what later in Africa one line of these led on to human-like apes. In the last half million years these came more and more to resemble ourselves, living in wooded African grasslands in small tribal units, until some hundred thousand years or so ago they began to disperse throughout the world, with eventually one species, homo sapiens, dominating. With exceptional social skills and intelligence, we became numerous and widespread, and ten thousand years ago population pressures led to agriculture, cities, organized religion, commerce and industry, and here we are.

At the genetic, bacterial, plant and insect levels completely different histories might be told.

What drives this constant change and fluctuation in the make-up of species? Conditions on the planet change, including competitors, predators and prey. Sexual selection and adaptive pressures mean that small variations can cause the genetic stock of a species to drift, just as people breed dogs or horses to bring out certain traits and discourage others, except that there is no overseer to the process. If times are lean and food is scarce, as in periods of drought, then smaller, lighter creatures will tend to survive more easily, and so the entire species on average gets lighter and smaller. On the other hand if food is plentiful, then it may be more advantageous to be large and strong, and the entire species becomes larger and stronger. Changes occur as a result of the pressures of the environment through the process of natural selection and sexual choices, and operate in a large scale statistical fashion that is difficult to appreciate close-up. Even a small advantage or inclination in one direction will result, after enough generations, in significant changes.

During the last few decades, evolutionary theory has made major inroads into anthropology, sociology and psychology. It is becoming increasingly clear that many aspects of modern human behavior are direct consequences of traits our forebears acquired during those hundreds of thousands of years as hunter-gatherers on the African savannah, and those millions of years before that as proto-apes. Other deeper aspects of what we are were laid down much earlier, during the hundreds of millions of years as small rodent-like animals under the feet of the dinosaurs.

A few ruffles

Although evolution is an established scientific fact, there are aspects of it that still need to be understood more clearly. There are gaps in the fossil record, but one would expect that, given the vast numbers of years involved. There are competing theories about whether evolution takes place more or less continuously, or rather in spurts. There are differences of opinion as to the level at which evolution takes place—are we just machines that are exploited by those tyrannical genes, or are genes just some chemical markers that we utilize to pass on our traits? Although modern theory favours the former explanation, I doubt that the issue has been settled yet.

There is an important question about whether the complexity observed in

nature can be adequately explained given the time available. The famous example of the eye, which Darwin himself pondered, still requires a more detailed argument. One needs to *quantify* the amount of complexity in such an organ, and demonstrate that with reasonable estimates of rates of mutation and selection adaptation, there has been enough *time* for its development without resorting to postulates of positive feedback or some other external guidance. Of course half a billion years is a long time, but is it long enough to get from next to nothing to an eye or the even more remarkable human brain? This is both a *biological* and a *mathematical* issue, and a very complicated one, but the argument still needs to be made convincingly. My own guess is that some positive feedback is indeed necessary, but that such an effect has a completely scientific explanation.

Another weakness is that some arguments of modern evolutionary theory have the whiff of ‘just-so’ theories. In other words, although they sound plausible enough, there is often a convincing argument in an opposite direction.

Why, for example, do department stores have dozens of counters pandering to *women’s* cosmetics, and virtually none for *men*? On evolutionary grounds we could argue that just as in nature it is the male birds that are brightly coloured, the male antelopes with dramatic headgear, the male lions with extravagant manes and so on, the sexual roles of men and women suggest that a man should be constantly pruning and decorating himself to appear attractive. A woman, confident of her importance as the bearer of children and the essential chooser of males, should be largely indifferent to her own attractiveness.

Similarly on evolutionary grounds one might think that homosexuality (say where males are inclined to have sex with other males) should be improbable, since any male oriented this way would not pass on his genes, and so the trait would be exterminated almost as soon as it appeared.

You might also argue that people of different races should find it impossible to live harmoniously with each other, because in the distant past any male from a different racial group would have been at best a competitor, and at worst a potential death threat. But modern society shows that, with some deep seated hostilities aside, we *are* capable of living peacefully, and even happily, with others not from our own background or race.

Such questions no doubt can be answered, but then these answers can also be challenged, and it seems that it is more a matter of debating skill rather than evidence which decides the issue.

The Platonist world of mathematics

Our physical world of stars, planets, plants and animals runs parallel to an abstract world of mathematics which is also full of a rich and varied assortment of objects. There is probably as much complexity and depth in this mathematical universe as in our physical one, and arguably much greater coherence and beauty, at least once you have learnt how to read it. This point will seem strange and perhaps even unbelievable to those readers who only remember from

their school math some rather dull arithmetic, difficult geometry and jumbles of trigonometric laws. Sadly, the lovely world of higher mathematics is witnessed only by very few.

Mathematical objects often parallel physical objects or processes. Sometimes connections appear long after the mathematical objects have been studied for aesthetic reasons by mathematicians, and then end up playing useful roles in computer science, engineering, physics and chemistry, and these days increasingly also in biology and social sciences. Most mathematicians develop an intuition that suggests that the mathematical objects they study are in some deep way *independent of the physical universe*, yet closely connected to it. This idea goes back to Plato, who held the extreme view that the physical aspect of our everyday reality is in truth only an imperfect reflection of the more fundamental *ideal* world of mathematical forms.

What kind of objects, or structures, are we talking about here? The easiest examples are *numbers*, like natural numbers such as $1, 2, 3, \dots$, or fractions, such as $2/3$ or $5/4$, *operations*, such as addition and multiplication, basic geometrical *shapes*, such as the circle or a triangle. There are *sequences*, such as the regular one of square numbers $0, 1, 4, 9, 16, \dots$ or the erratic one of prime numbers $2, 3, 5, 7, 11, \dots$, *patterns* like the tessellation of the plane by regular hexagons, and *transformations* such as reflections, rotations and dilations. There are *curves* like the ellipse, cycloid or bell curve, *surfaces* such as spheres, hyperboloids and tori (inner tubes), *regular solids* such as the icosahedron, dodecahedron or the four dimensional 120-cell, *lattices*, *graphs*, *symmetry groups*, *fields*, *Latin squares*, *operator algebras*, *varieties*, *dynamical systems*, *differential equations* and so on and so on.

Such a list gives only an abstract hint of the richness of the world of mathematics, so let's have a better look at something specific.

The Stern Brocot tree

The **Stern Brocot tree**, whose first few layers are shown in Figure 1, was discovered independently by a German mathematician and a French clockmaker in the middle of the nineteenth century. It is a remarkable pattern that ought to be more widely appreciated.

The tree is obtained by starting with the two 'fractions'

$$\frac{0}{1}, \frac{1}{0}$$

and at every step inserting the mediant of two adjacent fractions, where the **mediant** of a/b and c/d is defined to be the fraction

$$\frac{a+c}{b+d}$$

I remind the reader that this is definitely *not* the way we usually add fractions!

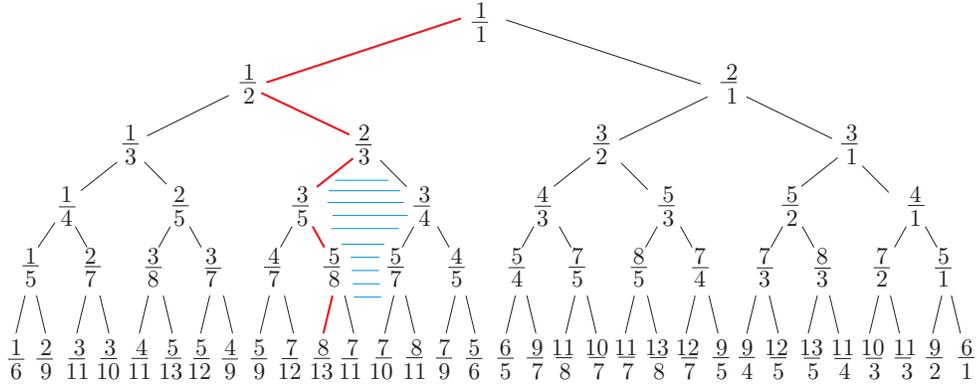


Figure 1: First six layers of the Stern Brocot tree

At the first step we get the sequence

$$\frac{0}{1}, \frac{1}{1}, \frac{1}{0}$$

at the second step

$$\frac{0}{1}, \frac{1}{2}, \frac{1}{1}, \frac{2}{1}, \frac{1}{0}$$

at the third step

$$\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}, \frac{3}{2}, \frac{2}{1}, \frac{3}{1}, \frac{1}{0}$$

and at the fourth step

$$\frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{2}{1}, \frac{5}{2}, \frac{3}{1}, \frac{4}{1}, \frac{1}{0}$$

You can easily continue the pattern. Another way to go from one step to the next is to first of all to each fraction add the numerator to the denominator—this gives the first half of the next step, while adding the denominator to the numerator gives the second half of the next step, with the two halves overlapping at the middle fraction $1/1$.

Ignoring the initial $0/1$ and $1/0$, the successive sequences may be recorded in a binary tree, where below each fraction a/b are the two new fractions which are introduced to the left and right of a/b at the step directly after a/b is introduced. This is shown in Figure 1.

The main feature of this Stern Brocot tree is that *every fraction in reduced form* appears in it in exactly one spot. There are other pleasant facts too: for example if you add up the inverses of the products of both numerators and denominators along any row, you always get 1. Along the third row for example you get

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{1}{3} = 1.$$

Each row contains the reciprocals of each of its elements. By proceeding zig-zag down the tree, first to the left, then to the right, then to the left and so on, we see the famous Fibonacci sequence

$$1, 1, 2, 3, 5, 8, 13, \dots$$

occurring in both numerators and denominators. The Stern Brocot tree separates the plane under it into infinitely many regions, one for each node of the tree. The region associated to the node $2/3$ is shown. Each of the nodes a/b on the boundary of this region has the property that $2b - 3a = \pm 1$. In particular, neighbours a/b and c/d in the Stern Brocot tree satisfy

$$ad - bc = \pm 1.$$

If we are at some node on the Stern Brocot tree, is there an easy way to know which two nodes are underneath it? The key is to look at another variant of the Stern Brocot tree. In this version, the top node is the two by two matrix

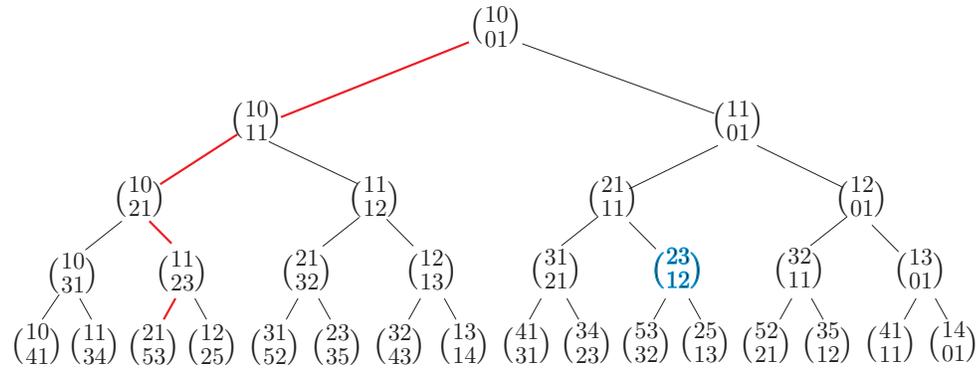


Figure 2: Matrix form of the Stern Brocot tree

(just an array of numbers):

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

To the left and down of I , we put the matrix formed by adding the *right* column of I to the *left* column. To the right and down of I , we put the matrix formed by adding the *left* column of I to the *right* column. This pattern involving these two operations, one to the left and one to the right, is repeated for all subsequent entries, giving Figure 2. To go from this matrix form of the Stern Brocot tree to the fraction form is easy: just add the columns of the matrix, and interpret the resulting column as a fraction. So for example the indicated entry corresponds to the column

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

so to the fraction $5/3$. In terms of these matrices, finding the two subsequent entries below any given one is easy; just add one column to the other, first right to left, then left to right. For those with an understanding of matrix multiplication, these two operations amount to multiplying (on the right) by the two matrices

$$L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \quad \text{and} \quad R = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

respectively. The path indicated in red is thus the cumulative product

$$LLRL = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}.$$

Ford Circles

In 1938 Lester Ford published in the *American Mathematical Monthly* a remarkable fact about rational numbers and certain circles in the plane, which turned out to be closely connected to the Stern Brocot tree.

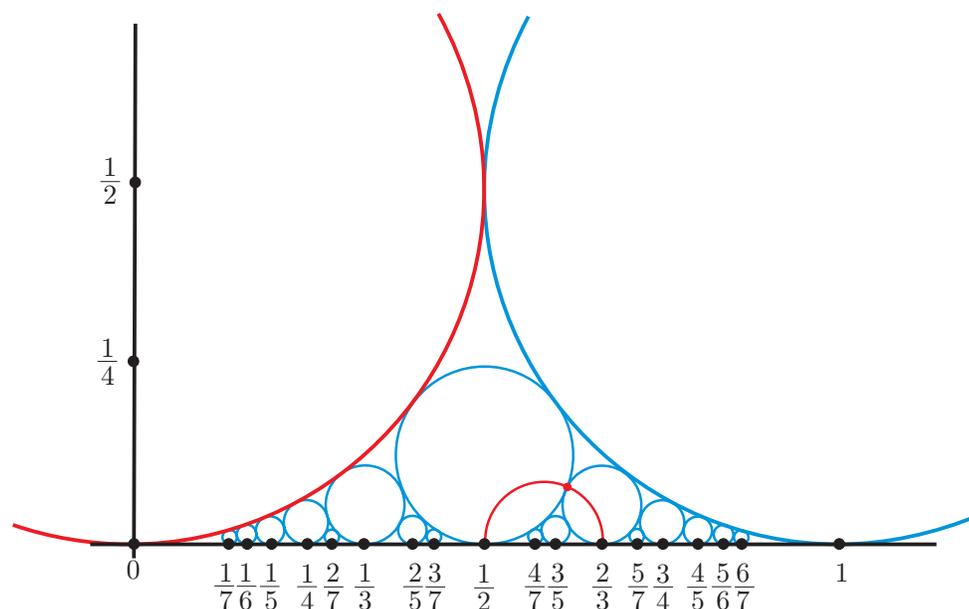


Figure 3: Ford circles

To each positive fraction p/q in reduced form, he introduced what is now called the **Ford circle** $C(p/q)$ which is tangent to the horizontal axis, touches it at the point p/q , and has diameter

$$d = \frac{1}{q^2}.$$

For example, the Ford circles above the integers $0, 1, 2, 3$ and so on are all of diameter 1, those above the fractions $1/2, 3/2, 5/2$ and so on are all of diameter $1/4$, and those above the fractions $1/3, 2/3, 4/3$ and so on are all of diameter $1/9$, and so on.

Those Ford circles above points in the interval 0 to 1 are shown in Figure 3. A fascinating fact about Ford circles is that *they are all disjoint*, except at points where they are tangent. This is despite the fact that there is one such circle above *every rational point* on the horizontal axis. Two Ford circles are tangent precisely when the two fractions are *neighbours in some Stern Brocot sequence*, which happens precisely when the corresponding regions in the Stern Brocot tree are adjacent. For example the circles $C(1/2)$ and $C(2/3)$ are tangent. Stated arithmetically, $C(a/b)$ and $C(c/d)$ are tangent precisely when

$$ad - bc = \pm 1.$$

When two Ford circles are tangent, their point of contact lies on a circle whose diameter passes through the corresponding fractions on the horizontal axis, so is perpendicular to it. This is shown in Figure 3 for the contact between $C(1/2)$ and $C(2/3)$. Such a picture is closely related to the upper half plane model of hyperbolic geometry, where the Ford circles are examples of *horocycles*, and the circles with diameters along the horizontal axis are *geodesics*.

To see the connections with the Stern Brocot tree, we remove the Ford circle $C(0/1)$ and get a pattern of circles stretching endlessly along the horizontal axis to the right, and between any consecutive integers get the same pattern of circles tangent to each other, as in Figure 4. Now each of these circles C is touching an infinite sequence of other circles. If we start with the circle $C(1)$ and move to the two largest circles tangent to C and to the left and right of it, we get $C(1/2)$ and $C(2/1)$. If we continue to move at every step to the largest tangential circle both to the left and right of whatever circle we are on, we get exactly the Stern Brocot tree. This is shown in Figure 4.

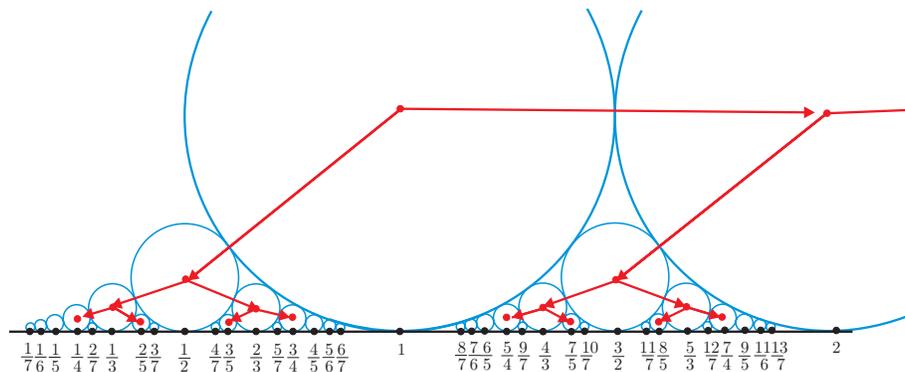


Figure 4: Ford circles and the Stern Brocot tree

All the Ford circles which are tangent to a given one form an infinite loop of

circles which can be identified with a region cut out by the Stern Brocot tree. For example the circles tangent to $1/2$ form two sequences, namely

$$\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \dots \quad \text{and} \quad \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \dots .$$

Visible points and wedges

There might be one nagging point about the Stern Brocot tree that is bothering you. We started the whole discussion with the two ‘fractions’ $0/1$ and $1/0$, which never actually made it into the picture. But the second one, $1/0$ is *not a fraction!* That means we were cheating, and in mathematics cheating inevitably leads to loss of understanding. That little nagging point is telling us that we have not yet comprehended things in the way we ought to.

Resolving this gives a quite different connection between the Stern Brocot tree and planar geometry, involving very special triangles which we might call *wedges*, and which I believe is a new point of view.

The setting is the positive quadrant consisting of all points $[x, y]$ where x and y are positive (greater than or equal to zero). Those points $[a, b]$ with a and b natural numbers are called **integral points**. An integral point $P \equiv [a, b]$ is called **visible** if it can be seen from the origin $O \equiv [0, 0]$, in the sense that the line segment between O and P does not pass through any other integral points. Algebraically this amounts to the condition that the natural numbers a and b have no common factors greater than one. So we almost get an identification between visible integral points $[a, b]$ and fractions a/b , except that the visible point $[1, 0]$ does not correspond to a fraction, since $1/0$ is not a fraction.

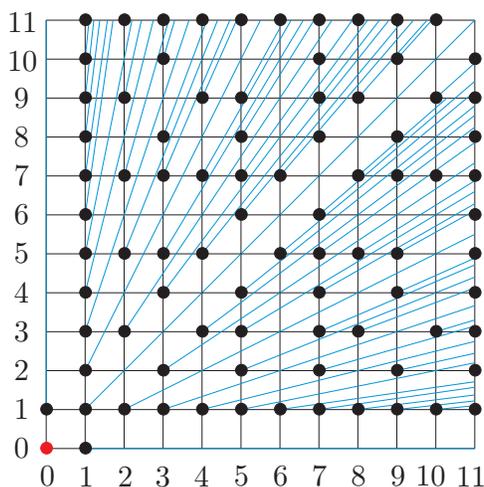


Figure 5: Visible points and their shadows

You can think of the origin as a tiny sun, and each visible point as a planet. Each planet will cast a shadow behind it, and those planets that are not in any other planet’s shadow are visible, as in Figure 5.

The mediant operation between fractions has a simple geometrical meaning: it corresponds to *adding the corresponding visible points*. The Stern Brocot tree may be seen geometrically by ignoring the points $[1, 0]$ and $[0, 1]$, starting with $[1, 1]$ and joining two visible points as in Figure 6.

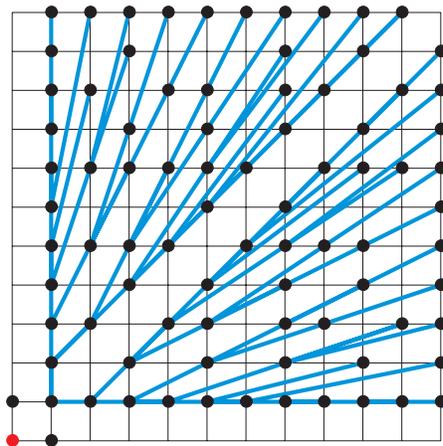


Figure 6: The planar form of the Stern Brocot tree

To the matrix

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

in the matrix form of the Stern Brocot tree we associate the triangle with vertices $[a, b]$, $[c, d]$ (called the **base vertices**) and $[a + c, b + d]$ (the **apex**).

Let’s call such a triangle a **wedge**—each has area exactly $1/2$. Wedges are hard to draw on account of their length and thinness, and perhaps this explains why this pattern seems not to have been considered before. Some of the less anorexic wedges are shown in Figure 7, in particular the wedge with vertices $[2, 1]$, $[5, 3]$ and $[7, 4]$.

So the original Stern Brocot tree can now be reinterpreted as the tree of *apexes of wedges*, and the matrix form of it can be reinterpreted as the collections of *bases of wedges*. You might like to convince yourself that every visible point (whether integral or not) which is in the positive quadrant but *not* in the darkened triangle Δ with vertices $[0, 0]$, $[1, 0]$ and $[0, 1]$, is contained in some wedge (perhaps more than one if it lies on an edge or is a vertex). So the infinite pattern of all wedges, together with Δ , ‘fills out’ the visible positive quadrant.

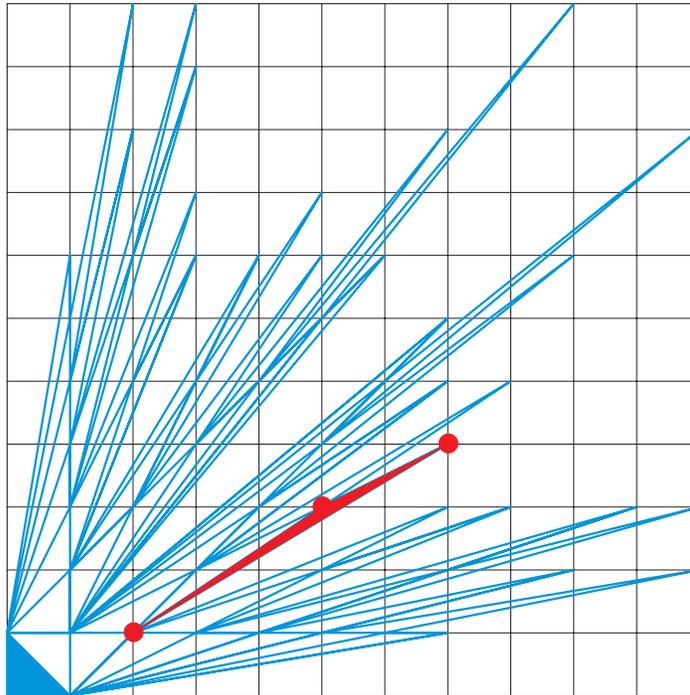


Figure 7: Wedges filling the visible positive quadrant

Discovery or creation?

There is an age-old philosophical question that asks: are mathematical patterns *discovered* or *created*? In my opinion, this is a no-brainer. Although we can pin down precisely when Lester Ford introduced the idea of Ford circles, the actual phenomenon has existed as part of the nature of reality long before the earth came into existence. It is built into the world, probably even more so than any physical, chemical or biological law. We mathematicians who research new areas continually find that the mathematics we study is much more clever, serious and interesting than we are, and resign ourselves to a humble kind of reverence towards the subject.

So there certainly *appears* to be a vast amount of intelligence encoded or built into the mathematical framework of the universe. Are phenomenon like the Stern Brocot tree or the pattern of Ford circles subject to some kind of evolutionary processes? I very much doubt it. Have they changed over time? Almost certainly not. But where do such structures and pattern *come from*? Who or what put them there? Is there behind this world of ours a richer world?

Such questions may have answers that will never be accessible to us, but that does not mean we should not occasionally ask them. Speculation about the higher world of a possible ‘creator’ is necessarily vague, but who knows if

and when future discoveries might provide more hints.

Some may argue that mathematical laws are not an aspect of this particular universe, but are necessary in ‘any possible universe’—that even if a ‘creator’ wanted to build a world with different mathematical laws, logic would prevent him (or her, or it). This is possibly correct, but it is also possibly incorrect. Our difficulty in imagining alternative mathematical realities doesn’t imply that they couldn’t exist. There are bounds to what we may know using logic and observation, and limits to the depth of the explanations we can arrive at. One reason is simple enough—we are *part of the world we study, not independent of it*. In fact we are so much a part of it that we may not have any choice in what we investigate, or think, or say, or do.

The end of free will

It seems increasingly likely from recent neurological, psychological and physical research that our traditional beliefs in our capacity for choice and free will are misplaced. Scientists are peeling back the complexities of the human brain and revealing the neurological and chemical aspects and origins of memories, feelings, behavior and thoughts. There now appears a rather continuous spectrum from us down to one-celled creatures, with no good reason to believe that at such and such a point free will stops and automatic processes take over.

The ticklish reality, my friends, is that *we are very likely all automatons*. Some species are bigger and more complicated automatons than others. While large and elaborate creature/machines like us can appreciate the clockwork aspects of much simpler animals, it is more difficult for us to comprehend our own deterministic natures. Yet throughout history thoughtful people have come to the conclusion that our destinies are predetermined.

Our perception of ourselves as free agents is an illusion—we think we are making decisions when in fact we are only *witnessing* complicated, but ultimately mechanistic, chemical and neurological interactions between different parts of our brains. Our increasingly powerful computers will also soon start to confront us with our own predictability, by establishing benchmarks for complexity that our conscious selves have no hope of equaling—the vast majority of our brain power is devoted to subconscious monitoring and processing.

Einstein showed with his special and general theories of relativity that the seemingly uniform march of time that we take for granted is largely an illusion due to the fact that we move around so slowly. Instead of viewing the world as a three dimensional space evolving steadily through time, Einstein and the mathematician Hermann Minkowski taught us to see a *four dimensional space-time* in which the division between space and time is ambiguous. Different observers travelling relative to each other observe different rates of the passage of time, leading to many surprising conclusions discussed in books on this subject.

I believe that a reasonable conclusion from our increasingly bio-mechanistic understanding of thought, feeling and consciousness—together with Einstein’s insights—is that *the world is already in place in its entirety*. The passage of

time, and the resultant dynamical aspect of the universe, is a mirage. We are complex neuro-machines, acting out our individual roles which have been preordained from the beginning.

There are scientists that have argued that quantum mechanics protects us from such deterministic views. But I would suggest that the impossibility for us to determine the future (which I fully accept) does not prevent the future from being already laid out. Naturally we cannot generally predict what will happen—that is not what determinism means. It means rather that *there is only one possible future*, or to put it into a relativistic setting, the *four dimensional space time is already established in its entirety*.

I predict this view will become the scientific orthodoxy later in this century, naturally to the great consternation of ordinary people, for whom the conceit of being in control of at least their own thoughts and actions will be hard to give up. This will have major philosophical, ethical and legal implications. It will also force physicists to recast their theories into an ‘observer-free’ form—the ‘collapse of the wave function’ idea involving an independent free thinking observer will have to be replaced by something more holistic, in which the ultimate unity of all things is an essential part of the picture. Modern physics has been moving in this direction for some time, but more remains to be done.

From this perspective, what seems to a biologist as the twist and turns of random events—the extinction of a species, the adaptation that leads to world dominance, the chance impact of a comet—have an alternative formulation as simply interesting aspects of a story that has *already been written*—an idea of a ‘Book of Destiny’ not without parallels in scriptures. Somehow the world in which we live has been created in its entirety, including the mathematical and physical laws that we uncover. Whether an external God is subject also to this inevitability is a question, but there is no need to believe so.

To make the case another way, imagine a magnificent medieval tapestry along a long hall of a castle, which you are only capable of seeing one centimeter at a time—say a single vertical slice that moves slowly to the right, always only revealing a fraction of the total picture. What you would see is the evolution of patches of colour and lines. Perhaps you could formulate theories that would allow you to guess what the ‘future’ would show. From the one dimensional perspective a two dimensional tapestry may appear as a dynamical process, an unfolding. But to the privileged eye (of the creator?) the picture does not move, and only in its entirety does it reveal its true meaning.

When stated in the form: *things which are successful at replicating themselves will become more numerous*, evolution becomes less of a biological law and more of an *economic* and *mathematical* one. While it has clearly played an important role in the history of life on earth, it may prove that there are not a lot of opportunities in the larger universe for structures to reach the complexity required for the possibility of replication. In this case, evolution as a vital ingredient for an understanding of the origin of the cosmos might well be over-rated. Evolution is in any case more an *aspect* to the overall pattern rather than an *explanation* of it.

Scientific types ought to be a bit more open-minded about things which

border on the unknowable. Dramatic denials of God or any kind of cosmic intelligence by biologists, esteemed as they may be in their own communities, seem to me to be almost as unjustifiable as the emphatic assertions of religious extremists, or the overly confident descriptions of the first trillionth of a second by Big-Bangers.

The deepest scientific question is not ‘How does the universe work?’ It is rather ‘Why is the universe here?’ My guess is that study of the chemical, physical and cosmological laws is insufficient to unravel this greatest of mysteries. The sciences have to be augmented with an *older* and *wiser* intellectual tradition. Let’s not view the world around us only as some soup of particles, molecules and creatures, evolving in a complicated but ultimately pointless fashion. There is also *mathematics*, which shows a truer beauty than that of which the poets sing, and a higher, divine reality far above the daily grind.