

We describe recent results on the evolution associated with degenerate second-order operators with real measurable coefficients in divergence form on  $\mathbf{R}^d$ , i.e., operators of the form

$$H = - \sum_{i,j=1}^d \partial_i c_{ij} \partial_j$$

where  $\partial_i = \partial/\partial x_i$ . The coefficients are assumed to satisfy  $c_{ij} = c_{ji} \in L_\infty(\mathbf{R}^d)$  and  $C = (c_{ij}) \geq 0$  almost everywhere. There has been an enormous amount of work under the assumption that the coefficients  $c_{ij}$  are smooth and satisfy the non-degeneracy condition  $C \geq \mu I > 0$  and the first part of the talk will review salient features of this work. We briefly describe the situation for strongly elliptic operators with smooth coefficients and the Nash–De Giorgi theory with measurable coefficients. Then we summarize some of the global features of operators constructed as the sum of squares of vector fields satisfying a uniform version of the Hörmander condition. Finally we turn to the recent results for operators with non-smooth coefficients satisfying the positivity condition  $C \geq 0$  or the stronger subellipticity condition

$$H \geq \mu \Delta^{1-\gamma} - \nu I$$

with  $\mu > 0$ ,  $\nu \geq 0$  and  $\gamma \in [0, 1)$  where  $\Delta$  is the usual Laplacian.