We study spectral multipliers of right-invariant sublaplacians with drift \( \mathcal{L} \) on an amenable, connected Lie group \( G \). The operators we consider are self-adjoint with respect to a measure \( \chi \, d\lambda \), whose density with respect to the left Haar measure \( d\lambda \) is a nontrivial positive character of \( G \). We show that if \( p \neq 2 \), then every \( L^p(\chi \, d\lambda) \) spectral multiplier of \( \mathcal{L} \) extends to a bounded holomorphic function on a parabolic region in the complex plane, which depends on \( p \) and on the drift. When \( G \) is of polynomial growth we show that this necessary condition is nearly sufficient, by proving that bounded holomorphic function on the appropriate parabolic region which satisfy mild regularity condition on its boundary are \( L^p(\chi \, d\lambda) \) multiplier of \( \mathcal{L} \).