

I will report on recent advances which follow on from the proof of the Kato Square Root Problem. One result concerns the spectral theory of the Dirac-type operator $d + d_g^*$ on a compact Riemannian manifold M . It is that the positive and negative eigenspaces of the self-adjoint operator $d + d_g^*$ depend analytically on L^∞ changes in the metric g .

In joint work with Andreas Axelsson and Stephen Keith, we showed that some key ideas in the proof of the Kato problem can be applied to obtain quadratic estimates for perturbations $\Pi_B = d + B^{-1}d^*B$ of a Dirac-type operator $\Pi = d + d^*$ acting in $L^2(\mathbb{R}^n, \Lambda)$. The operator B is multiplication by an L^∞ matrix-valued function with uniformly positive real part. Our result has not only the Kato square root theorem as a corollary, but includes many results in the Calderón program such as the boundedness of the Cauchy operator on Lipschitz curves and surfaces.