Diffusion on locally compact ultrametric spaces

A. Figà-Talamanca

23rd May 2004

Abstract

Let G be a locally compact totally disconnected Abelian group. Then G may be given, in a canonical (but not unique) fashion the structure of an ultrametric space in such a way that translations by elements of G are isometries. Conversely a locally compact ultrametric space, with sufficiently many isometries, may be endowed with the structure of an Abelian group, which is identified with a subgroup of the isometries of the space. In this lecture we start with a locally compact totally disconnected Abelian group G which is not compact and is the union of a countable chain of compact subgroups. This implies that there exists a double chain $\{G_n : n \in \mathbb{Z}\}$ of compact subgroups of G, such that $\bigcap_{n=1}^{\infty} G_n = \{0\}$, $\bigcup_{n=-1}^{-\infty} G_n = G$ and G_n/G_{n+1} is a cyclic group of order a prime number p_n. This chain of subgroups allows us to define an ultrametric structure on G. The metric balls of the ultrametric space are the cosets $G_n + a$, with $a \in G$, and they may identified in turn with the vertices of a tree, with boundary $G \cup \{\infty\}$, the one-point compactification of G. Accordingly the double chain H_n of horocycles of the tree with respect to ∞ is identified with the cosets of G_n. In other words $H_n = \{G_n + a : a \in G\}$. In this lecture we will construct a class of diffusion processes on G, which are associated to random walks on the tree and their hitting distributions on the horocycles H_n. We will show that our construction includes all processes associated through the Lévy–Khintchine formula to an unbounded Lévy measure.