

## NOTES AND CORRESPONDENCE

**On the Stochastic Forcing of Modes of Interannual Southern Ocean Sea Surface Temperature Variability**

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## ABSTRACT

A simple linearized transport model of anomalous Southern Ocean sea surface temperature (SST) is studied to determine whether it can sustain anomalies of realistic amplitudes under a physically based stochastic forcing. As noted in previous studies, eigenmodes of this system with zonal wavenumbers 2 and 3 share key propagation characteristics with the SST anomalies associated with the Antarctic Circumpolar Wave (ACW). The system is solved on a grid that follows the path of the Antarctic Circumpolar Current (ACC) and is forced by a stochastic heat flux. The forcing is white in space and time and represents the advection of the mean SST gradient by high-frequency variations in the cross-ACC velocity, due to meso-scale eddy variability. The magnitude of the stochastic forcing is determined from a global eddy-permitting ocean model. Anomalous ocean surface velocity variability ( $8 \text{ cm s}^{-1}$ ) coupled to a mean cross-ACC SST gradient of  $0.8^\circ\text{C} (\text{latitude})^{-1}$  sustains anomalous interannual SST variability at low wavenumbers and amplitudes of the order of  $1^\circ\text{C}$ , consistent with those associated with the ACW. In the long-term mean, variance is broadly spread among low wavenumbers, in contrast to the dominance of one or two zonal wavenumbers in the ACW observations. It is found, however, that the model produces single dominant wavenumbers over individual periods of decades, suggesting that the apparent unimodal nature of the ACW may be an artifact of the short observational record used to infer it. Alternatively, it is shown that a nonisotropic forcing may also result in a stronger preference for particular zonal wavenumbers. It is shown that if the atmosphere at mid to high southern latitudes has an equivalent barotropic response to heating, then the resulting sea level pressure anomalies reproduce the phase relationship of the observed ACW. These results are consistent with the notion that a simple stochastically forced advection of SST anomalies can explain SST variability associated with the ACW to leading order.

**1. Introduction**

Observational evidence suggests the existence of modes of interannual variability in a number of the ocean's basins. The most studied and best understood of these is the ENSO phenomenon in the tropical Pacific, but modes of interannual to decadal variability have also been documented in the North Atlantic, the North Pacific, and the Southern Ocean. The most heralded mode of interannual variability in the Southern Ocean is the Antarctic Circumpolar Wave (ACW), first

observed in sea surface temperature (SST), sea level pressure (SLP), surface winds, and sea ice extent by White and Peterson (1996) and in sea surface height (SSH) by Jacobs and Mitchell (1996). The ACW has been characterized from observations as an eastward-propagating set of phase-linked anomalies in the above variables, with dominant wavenumber-2 spatial pattern and a period of around 4 yr. Owing to the limited amount of data available in the analysis of White and Peterson (1996), the robustness of the ACW characteristics has been a topic of some debate. Several numerical models have been shown to exhibit ACW-like variability in SST and sea surface density (e.g., Christoph et al. 1998; Weisse et al. 1999; Bonekamp et al. 1999; Cai et al. 1999), though with a number of differences to the White and Peterson (1996) analysis.

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While ENSO is a true coupled mode of the atmosphere–ocean system, the generating mechanism for extratropical modes of variability remains less clear. The ACW observations of White and Peterson (1996) show a fixed phase relationship between SST and SLP, suggestive of a coupled mode. Qualitatively plausible feedback mechanisms have been proposed that are consistent with the observed phases, in which high (low) SLP anomalies drive positive (negative) SST anomalies, which in turn reinforce the original SLP through anomalous heating. The studies of Qiu and Jin (1997), Talley (1999), and Baines and Cai (2000) found that self-sustaining coupled modes with the phase relationship of the ACW observations exist in simple coupled models of the Southern Ocean. In the case of Qiu and Jin (1997) and Baines and Cai (2000), the fastest growing modes were found to share many of the characteristics of the ACW from White and Peterson (1996). Talley (1999) concluded that an ocean with a Sverdrup response to wind stress forcing coupled to an atmosphere with a Sverdrup response to heating-induced vertical advection best matches the observed phase relationship. In addition, White et al. (1998) demonstrated that atmosphere–ocean coupling was necessary to sustain ACWs that match the observed propagation characteristics in a coupled model of the global lower troposphere and upper ocean. Baines and Cai (2000) showed that large-scale low-frequency forcing is required to sustain the coupled component of the ACW.

There exists doubt, however, that atmospheric heating in the extratropical oceans is sufficiently strong to sustain true coupled modes and that the observed variability is more likely dominated by atmosphere-to-ocean forcing. Realistic global coupled models have produced ACW-like responses in Southern Ocean SST (Christoph et al. 1998; Weisse et al. 1999; Bonekamp et al. 1999; Cai et al. 1999), but without the observed wavenumber and phase relationship with SLP. The interannual SST variability in a number of these models was shown to be well described to leading order by simple forced advection models, and hence the existence of a fully coupled mode was not necessary to explain the interannual variability. These studies also found that the model SST variability was predominantly determined by local atmospheric forcing rather than atmospheric teleconnection. Qiu and Jin (1997) also suggest this to be the case for the ACW from an analysis of the propagation of anomalous atmospheric pressure from the Tropics to the midlatitude Southern Hemisphere. Cai and Baines (2001) and Venegas (2003) show, however, that the observed wavenumber-2 ACW variability is predominantly driven remotely by ENSO through forcing of the Pacific South

American (PSA) pattern, while the lesser wavenumber-3 component is due to local coupling. Thus Cai and Baines (2001) suggest that the wavenumber discrepancy between coupled models and observations may be a result of the weakness of ENSO events in the models.

A number of authors have shown that a coherent integrated ocean response is possible under essentially stochastic forcing, bypassing the notion of exponentially growing modes (e.g., Saravanan and McWilliams 1998; Hasslemann 1976; Frankignoul and Reynolds 1983). A related approach has been to understand ocean/atmosphere variability as modal in nature, but sustained by stochastic forcing rather than through linear instability of the modes themselves (e.g., Jin 1997; Penland and Sardeshmukh 1995; Griffies and Tziperman 1995; Kleeman and Moore 1997). In this note we use the latter approach to investigate the response of a simple model of SST variability to stochastic forcing, under the hypothesis that interannual SST fluctuations in the Southern Ocean may be described in part as uncoupled stochastically forced damped linear modes. As the high-frequency processes that form the stochastic forcing considered here are typically underrepresented in coupled GCMs, we do not set out a priori to reproduce or explain ACW behavior within coupled climate models. Rather, we seek to examine the possibility that stochastic forcing can excite ACW-like modes of variability in a simple channel model of Southern Ocean SST. The mechanism discussed in the following may help account for the relative weakness of the ACWs simulated in these models.

## 2. A simple model of interannual SST variability

In this note we consider a simple linearized advection model for interannual Southern Ocean SST anomalies. The unforced evolution of SST may be written as

$$T_t = -\mathbf{U} \cdot \nabla T + S(T), \quad (1)$$

where  $T$  is the total SST;  $\mathbf{U} = [U \ V]$  is the surface velocity vector,  $\nabla = [\partial_x \ \partial_y]$ , where  $\partial$  represents the derivative operator with respect to the subscript variable, and  $x$  and  $y$  are the zonal and meridional coordinates, respectively; and  $S(T)$  contains any other internal sources or sinks of heat, such as vertical advection or mixing. In the following we consider  $S(T) = -\lambda_o T - \kappa_o \nabla^2 T$ , where  $\lambda_o$  is a thermal damping or feedback coefficient, and  $\kappa_o$  is an eddy diffusivity. If a standard linearization is performed, in which  $T$  and  $\mathbf{U}$  are decomposed into mean and anomaly components (represented by overbarred and primed variables, respectively), and second-order terms are ignored, the evolution of SST anomalies  $T'$  can be written as

$$T'_t = -\bar{\mathbf{U}} \times \nabla T' - \mathbf{U}' \times \nabla \bar{T} - \lambda_o T' - \kappa_o \nabla^2 T'. \quad (2)$$

In addition, as the mean flow in the Southern Ocean is dominantly zonal, we assume  $\bar{\mathbf{V}} = 0$ , and for now ignore advection by the anomalous ocean velocities, yielding

$$\begin{aligned} T'_t &= -\bar{U} T'_x - \kappa_o \nabla^2 T' - \lambda_o T' \\ &= -(\bar{U} \partial_x - \kappa_o \nabla^2 - \lambda_o) T'. \end{aligned} \quad (3)$$

The operator

$$\mathbf{A} = -(\bar{U} \partial_x - \kappa_o \nabla^2 - \lambda_o) \quad (4)$$

is called the tangent linear operator. Clearly it is the properties of  $\mathbf{A}$  that determine the system's behavior.

It has been noted in earlier studies (Christoph et al. 1998; Weisse et al. 1999) that simple uncoupled heat transport models of SST anomalies similar to (3) are capable of sustaining an ACW-like signal in SST when a continuous forcing of certain spatial and temporal characteristics is applied. Christoph et al. (1998) showed that either a standing or propagating forcing of fixed wavenumber and frequency could sustain propagating SST anomalies of the same wavenumber. The result was generalized in Weisse et al. (1999) to show that a forcing of fixed zonal wavenumber that varied stochastically in time could produce ACW-like-propagating SST anomalies. It is demonstrated in section 4 that this result may in fact be generalized further—such models can sustain an ACW-like response for purely stochastic forcing in space and time and in fact can reproduce the amplitude of SST anomalies associated with the ACW.

### 3. Eigenmodes of the zonally uniform model

Assuming solutions for  $T$  of given zonal wavenumber structure  $e^{ikx}$ , but which are uniform, then

$$\mathbf{A} = -ikU + \kappa_o k^2 - \lambda_o. \quad (5)$$

Note that the assumption of no meridional structure is made for simplicity, as the results that follow are relatively insensitive to the inclusion of meridional structure of the form  $e^{ily}$  for reasonable values of  $l$ . The frequency and decay rates of a disturbance with zonal wavenumber  $k$  are given by  $kU$  and  $(\kappa_o k^2 - \lambda_o)$ , respectively. As a result, when (3) is forced with a particular zonal wavenumber  $k$ , variance would be expected to reside predominantly at the eigenfrequency  $kU$ . It has been noted previously that this relation matches the observed ACW frequency for typical values of  $U$  for the Southern Ocean combined with the observed zonal wavenumber-2 and/or -3 structure of the ACW, and this has been used as evidence that the

ACW SST anomalies are simply advected by the mean flow rather than propagating relative to it (e.g., White and Peterson 1996; Christoph et al. 1998).

The results of Christoph et al. (1998) and Weisse et al. (1999) confirm that the response of the simple advection model (3) predominantly takes the form of the modes that are excited by the forcing. In their cases, a wavenumber-3 mode was predominantly excited when a purely wavenumber-3 forcing was used, representative of the dominant Southern Hemisphere SLP pattern that potentially drives the Southern Ocean SST anomalies. The wavenumber-3 pattern is the main forcing of ACW-like variability in the coarse resolution coupled model of Cai et al. (1999), ENSO being largely suppressed by the model's coarse resolution. Here we investigate the possibility that other sources of forcing may be responsible for the locally forced interannual SST variability associated with the ACW. In particular, we consider sources of external forcing that are of much higher frequency than that of the ACW, and so may be approximated as stochastic processes. It is common practice, for example, to treat high-frequency wind stress variability as stochastic noise forcing for the more slowly responding ocean (e.g., Frankignoul and Reynolds 1983).

Thus we consider the possibility that modes with a range of zonal wavenumbers are excited concurrently, in which case it becomes necessary to consider the modal decay rates in order to predict which modes dominate the response. The expression for decay rate given above reveals that, for nonzero  $\kappa_o$ , modal damping increases quadratically with zonal wavenumber. Thus when all modes are equally excited, such as by a spatial and temporal white-noise forcing, the model would be expected to have a dominantly low zonal wavenumber response. The sustenance of these modes is demonstrated in the following section for a discretized version of (3).

It may be noted that neither the Christoph et al. (1998) nor Weisse et al. (1999) models include an eddy diffusivity term, so that for the simple models included in their studies the modes at each zonal wavenumber are in fact equally damped. The inclusion of a diffusivity, however, would not greatly alter their results, given that neither study was concerned with absolute or relative growth rates between the modes.

### 4. A zonally discretized model of SST evolution

Numerous studies have indicated that coherent variability can result when linear damped geophysical systems are stochastically forced (e.g., Farrell and Ioannou 1996; Kleeman and Moore 1997). It is demonstrated

below that interannual eigenmodes of (3) may similarly be sustained at significant amplitude by realistic levels of stochastic forcing. A plausible candidate to provide such forcing of SST is the uncoupled and uncorrelated variability of the atmosphere and ocean, through heat flux variations in the former, and anomalous horizontal and vertical advection of heat in the latter. Compared to the ACW, much of this variability is of high frequency and short spatial scale, and as a result may be considered essentially stochastic in time and space. Of these potential sources, the stochastic forcing considered here is chosen initially to simulate random variations in anomalous meridional ocean surface velocity but may equally correspond to forcing from high-frequency variations in Ekman pumping or atmospheric heat flux.

The model discussed thus far assumes homogeneity of the mean circulation. To include a greater degree of realism, an additional degree of freedom is added to the model. Rather than allowing only strictly zonal variability, we choose the new degree of freedom to be along the axis of the Antarctic Circumpolar Current (ACC), which is a more natural coordinate for advection of anomalies in the Southern Ocean. In other words, in this model we consider the evolution of SST anomalies driven by advection of the background SST field normal to the path of the ACC.

To obtain a well resolved and dynamically consistent approximation of Southern Ocean surface velocity and temperature fields, we use data from the Ocean Circulation and Climate Advanced Modelling (OCCAM) project, a 12-yr simulation of the global ocean circulation using a 1/4-degree primitive equation model (Webb et al. 1998). Mean and variability in the surface velocity and temperature fields were calculated from 4 yr of monthly OCCAM data for the region between 30° and 70°S. These fields were averaged into bins of size 4° latitude by 8° longitude, yielding data at 10 latitude and 45 longitude bins. The path of the ACC was determined by finding the latitude with largest eastward velocity at each longitude, giving an ACC path length of approximately 27 000 km. The mean along-path velocity was approximately 10 cm s<sup>-1</sup>, with a maximum of approximately 12 cm s<sup>-1</sup> at 140°W and a minimum of approximately 6 cm s<sup>-1</sup> at 124°E. This gives a circumpolar advection time scale of approximately 8.5 yr. The tangent linear operator was determined from

$$\mathbf{B} = -\mathbf{U}_p \mathbf{D}_p - \kappa_o \mathbf{D}_p^2 - \lambda_o, \quad (6)$$

where  $p$  is the along-path coordinate,  $\mathbf{U}_p$  contains the along-path velocity,  $\mathbf{D}_p$  is the finite difference along-path derivative operator,  $\mathbf{D}_p^2$  represents the second derivative along-path, and  $\mathbf{T}_t = \mathbf{B}\mathbf{T}$ . In this case  $\mathbf{T}$  is a

length 45 vector containing the SST anomaly at each bin along the ACC path, and  $\mathbf{B}$  is a 45 × 45 matrix.

The stochastically forced version of (3) takes the form

$$\mathbf{T}(t + 1) = \mathbf{T}(t) + \mathbf{B}\mathbf{T}(t)\Delta t + \xi(t), \quad (7)$$

where  $\Delta t$  is the time step, and  $\xi(t)$  contains the stochastic forcing. It may be noted that in (7) the forcing is independent of the system state, while in the real system it may be expected that the forcing, in this case through mesoscale ocean variability, is state dependent to some degree (e.g., Sura 2003; Gent and McWilliams 1990). Such multiplicative noise is not considered here. The values of the parameters used are  $U = 10 \text{ cm s}^{-1}$ ,  $\kappa = 10^3 \text{ m}^2 \text{ s}^{-1}$ ,  $\lambda_o = (3 \text{ months})^{-1}$ ; these being typical of values applied in the Southern Ocean. Here,  $\Delta t$  is chosen to be 1 week, and  $\xi(t)$  is generated by random sampling between  $-1$  and  $1$  and then scaled to have a standard deviation of  $\langle \bar{T}_n \rangle \langle u_n \rangle$ , where  $\langle \cdot \rangle$  signifies the along-path average, subscript  $n$  denotes the derivative normal to the ACC path,  $\bar{T}$  is the mean SST as above, and  $u$  is the standard deviation of anomalous cross-ACC surface velocity. In this case  $\bar{T}$  and  $u$  were determined from the OCCAM data, yielding  $\langle \bar{T}_n \rangle \approx 0.8^\circ\text{C} (\text{°lat})^{-1}$  and  $\langle u_n \rangle \approx 8 \text{ cm s}^{-1}$ . While the forcing is motivated by anomalous horizontal advection of the mean SST gradient by mesoscale eddies, the forcing may equally be interpreted as a stochastic vertical heat flux due to Ekman pumping or an atmospheric heat flux of appropriate magnitude. Southern Ocean surface temperature variability due to vertical flux processes was of the same order as that due to horizontal advection in the coupled GCM analyzed by Rintoul and England (2002). The forcing described above would equate to an anomalous wind stress curl of order  $10^{-7} \text{ N m}^{-2}$  coupled to a vertical temperature gradient of order  $10^{-2} \text{ }^\circ\text{C m}^{-1}$ , and for the latter case to an anomalous heat flux of order  $20 \text{ W m}^{-2}$ , each of which may be considered of reasonable order for the Southern Ocean. The stochastic forcing  $\xi$  is white in both space and time, thus representing forcing time and space scales that are considerably shorter than the ACW.

An ensemble of 100 simulations using (7) were run, each for a duration of 100 yr. A 20-yr segment of the system response to the stochastic forcing is presented in the Hovmoeller diagram of Fig. 1a. The data in this figure have been filtered with an admittance window of 2–7 yr, consistent with the analysis of White and Peterson (1996). The figure suggests the presence of large-scale eastward-propagating anomalies. While most individual anomalies are not truly circumpolar, many can be seen to persist for over 180° of longitude. The dominance of large-scale low-frequency disturbances in the

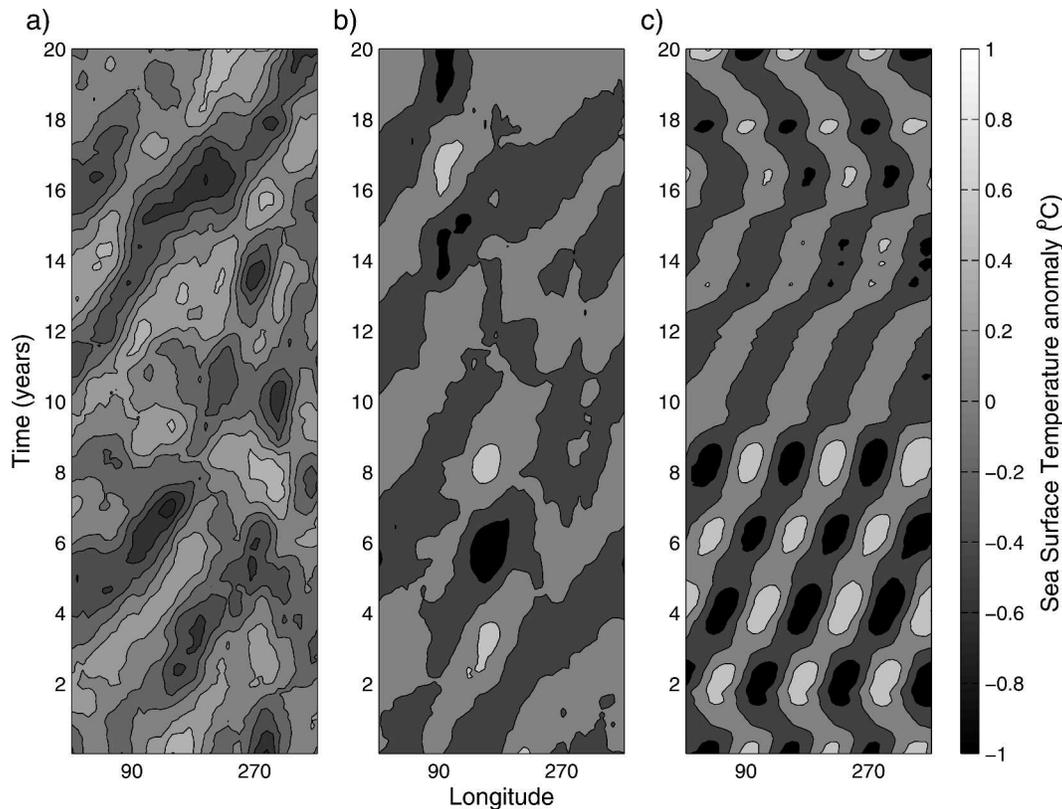


FIG. 1. Hovmoeller diagram of the response of the one-dimensional SST advection model to stochastic forcing. Shown is the response to (a) isotropic forcing, (b) anisotropic forcing proportional to the local SST gradient and cross-path velocity variability, and (c) random phase wavenumber-3 forcing. Data in each panel have been band-pass filtered to retain variability associated with periods between 2 and 7 yr.

model response is confirmed in Fig. 2, which shows the power spectral density as a function of along-path wavenumber and of frequency for the unfiltered ensemble simulations. Despite the fact that the forcing is spatially and temporally uncorrelated, the system variability resides predominantly in low wavenumbers and in frequencies  $<(2 \text{ yr})^{-1}$ . The amplitude of the filtered interannual SST anomalies is up to  $1^{\circ}\text{C}$ , of the same order as or slightly above those associated with the ACW from observations. Note that, being a linear model, the size of the response in (7) scales linearly with the forcing magnitude, while the structure of the response is amplitude independent. The forcing magnitude could thus be tuned to reproduce observed SST anomaly magnitudes exactly. However, the point here is that the simple model (7) can produce an ACW of significant amplitude under realistic stochastic forcing.

The response of this discretized system may be explained from eigenanalysis of the linear operator  $\mathbf{B}$ , as suggested in the previous section. Details of the least damped modes of  $\mathbf{B}$  are given in Table 1. The least damped mode simply corresponds to the uniform decay

of all anomalies at the thermal damping rate  $\lambda_o$ . The next least damped modes correspond to anomalies of increasing along-path wavenumber and decreasing frequency. For the modes of low-wavenumber the decay rates increase only gradually with wavenumber, while the frequencies of the first four wavenumbers are  $<(2 \text{ yr})^{-1}$ . From this it may be seen that, as suggested in section 3, the system response to stochastic forcing corresponds to the least damped modes in both wavenumber and frequency. The stochastic forcing excites all modes, but variance is maintained predominantly in the interannual, low-wavenumber modes, these being the least damped.

Thus in this simple model, coherent low-wavenumber interannual SST anomalies were sustained by a forcing that was random in both space and time. The model spectra differ from that of the observed ACW, however, in that no dominant wavenumber or frequency was seen—a result of the relatively small differences in the decay rates of low-wavenumber modes and the fact that in the ensemble mean they were evenly excited by the white-noise forcing. On shorter time

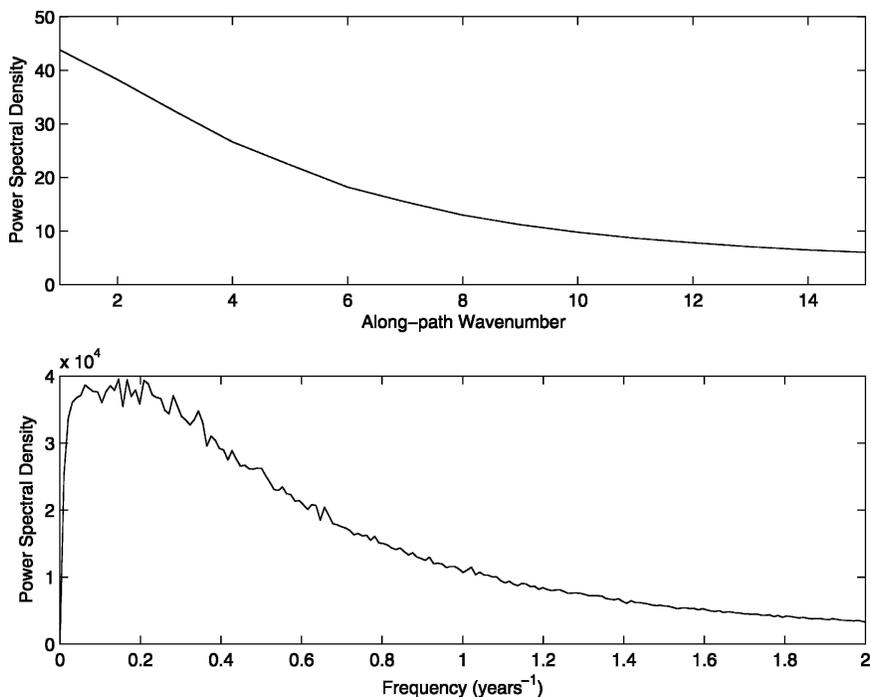


FIG. 2. Power spectral density of stochastically forced SST as a function of (top) along-path wavenumber and (bottom) frequency, averaged over an ensemble of 100 simulations of 100 yr each. Units of power spectral density are  $(^{\circ}\text{C})^2/\text{cpy}$ , where cpy = cycles per year.

scales (e.g., in some individual 100-yr simulations), dominant spectral peaks do appear at the low wavenumbers and interannual frequencies, so that it is only in the ensemble mean that the broadness of the spectrum is assured. This suggests the possibility that the 13 yr of SST data available for the analysis of White and Peterson (1996), despite appearing dominantly unimodal over this time period, may in fact be representative of a broad spectrum stochastically forced system such as the model studied here. That is, the apparent unimodal nature of the SST variability in the ACW observations may be an artifact of the shortness of the record. Similarly, Christoph et al. (1998) observed around 20% of their GCM simulations to have a wavenumber-2 structure consistent with the ACW observations, but in the ensemble average wavenumber 3 was equally important. The 2–7-yr bandpass filter employed by White and Peterson (1996) would also tend to increase

the unimodal appearance of the data by removing much of the variability from the higher wavenumbers and also from wavenumber 1. In Fig. 1a, application of such a filter helps isolate eastward-propagating wavenumber-2 anomalies in one individual 20-yr segment. While segments such as that shown in Fig. 1a are not uncommon in the ensemble, in the long-term mean the system has no particular preference to produce wavenumber 2, as shown in Fig. 2. It may be noted that Baines and Cai (2000) suggest that wavenumber 2 is the preferred pattern in the real system owing to this being the structure of the ENSO-induced PSA in southern mid- to high latitudes.

The forcing due to anomalous cross-path advection of the mean SST field considered thus far is of order  $10^{-7} \text{ } ^{\circ}\text{C s}^{-1}$ . As mentioned above, such a forcing could equally be interpreted as being a stochastic vertical heat flux due to Ekman pumping or atmospheric heat flux variations of reasonable order for the Southern Ocean. That is, stochastic isotropic forcing from any of these sources is capable of sustaining the model response presented above. Differences in the model response arise only when account is taken of the differing (nonisotropic) spatial structure of each forcing. For example, the OCCAM data suggest that anomalous cross-path SST advection is dominated by forcing south of Africa,

TABLE 1. Summary of characteristics of the least damped modes of the one-dimensional SST advection operator.

Mode number	1	2	3	4
Wavenumber	0	1	2	3
Period	—	8.6 yr	4.3 yr	2.9 yr
Decay time scale	1 yr	0.96 yr	0.84 yr	0.70 yr

where strong meridional SST gradients coincide with robust mesoscale eddy activity. To assess this, we apply a zonally varying forcing based on the local mean SST gradient and anomalous cross-path velocity variability in OCCAM. This forcing produces maximum anomalous SST south of Africa at the site of greatest forcing (approximately 90°E), gradually reducing in magnitude across the Pacific sector before increasing again east of the Drake Passage. A 20-yr segment of the model response to such a forcing is shown in Fig. 1b. Again, low-wavenumber SST anomalies of realistic amplitude are sustained, but in this case the response does not simply follow the modal decay rates. Wavenumbers 2 and 3 contain maximum variance in the long-term mean, as these modes are more strongly excited by this nonisotropic forcing than wavenumber 1. The resulting zonal modulation in SST, however, is not consistent with that observed in the ACW, whose amplitude is found to be relatively independent of longitude (e.g., White et al. 1998).

The well-documented dominant standing wavenumber-3 pattern of anomalous SLP over many time scales in the Southern Hemisphere (Mo and White 1985) suggests that anomalous atmospheric heat flux, Ekman pumping, and cross-path Ekman transport may each be expected to contain a significant wavenumber-3 component. Figure 1c shows a 20-yr segment of the evolution of the model when forced with a wavenumber-3 pattern that varies stochastically in time and phase. That is, the forcing is neither standing nor propagating. The forcing magnitude is the same as was considered above, corresponding to variations in vertical heat flux of order  $20 \text{ W m}^{-2}$ , Ekman pumping of order  $10^{-5} \text{ m s}^{-1}$ , or Ekman transport of order  $5 \text{ cm s}^{-1}$ . Figure 1c demonstrates that a random phase wavenumber-3 forcing sustains a generally eastward-propagating wavenumber-3 response in SST of order  $1^\circ\text{C}$ . The apparent westward propagation of phase seen from years 16 to 20 is in part an artifact of the contouring method but also reflects the fact that the random phase forcing can at times drag the phase of SST anomalies upstream. Note that such westward propagation is not seen in the observations. The GCM studies of Christoph et al. (1998), Weisse et al. (1999), and Bonekamp et al. (1999) find vertical heat flux to be the primary source of ACW-like SST anomalies, while the coupled climate model analysis of Rintoul and England (2002) suggests Subantarctic Mode Water temperature variability to be largely driven by meridional Ekman transport across the path of the ACC.

These results demonstrate that under white-noise isotropic forcing, dominance of one mode vanishes in the ensemble mean. While dominance of one mode can

occur at random over short time scales, long-term dominance of one mode is only possible when the forcing consistently projects more strongly onto that mode. Similarly, a dominant wavenumber-3 response can be achieved through an isotropic forcing that is harmonic at the resonant frequency of the wavenumber-3 SST mode (not shown). The response of the simplified model considered in Weisse et al. (1999) may be understood as the atmospheric forcing in their model preferentially exciting the wavenumber-3 mode of the SST advection operator.

## 5. Coupled SLP anomalies

The results presented thus far support the hypothesis that interannual SST anomalies associated with the ACW may represent the natural response of the Southern Ocean SST field to high-frequency forcing, rather than necessarily being part of a coupled mode. A key feature of the observations, however, is the apparent phase locking between a number of independent variables, most notably SST, SLP, and SSH, which suggests that some coupling is present. Previous studies have noted that coupled modes can exist with ACW-like characteristics and in fact can be the fastest growing of all system modes (Qiu and Jin 1997; Talley 1999; Baines and Cai 2000). In this final section, we explore a mechanism through which the stochastically forced SST anomalies considered to date can drive SLP anomalies with the observed phase relationship. For this purpose the SST model (3) is coupled to the diagnostic model for SLP anomalies of Qiu and Jin (1997).

The atmosphere is considered to be in heat balance, in which case the equation for anomalous atmospheric temperature  $T_a$  for a purely zonal mean temperature  $\bar{T}_a$  and wind field  $\bar{U}_a$  is

$$\bar{U}_a T_{ax} + v_a \bar{T}_{ay} + w_a \bar{T}_{az} = -\lambda_a T_a + b \lambda_o T, \quad (8)$$

with  $v_a$  and  $w_a$  as the meridional and vertical wind anomalies,  $\lambda_a$  as the atmospheric thermal damping rate, and  $b$  as the conversion coefficient for thermal forcing. In this model, the response of the atmosphere to the oceanic heat flux is parameterized as a change in the SLP anomaly  $p$  through

$$p = \gamma T_a. \quad (9)$$

That is, the atmosphere is assumed to have an equivalent barotropic response to surface heating. While the analysis of Baines and Cai (2000) brings the validity of this assumption into question, observations do provide some support for it. Analysis of 55 yr of monthly mean National Centers for Environmental Prediction (NCEP) data of SLP and surface atmospheric tempera-

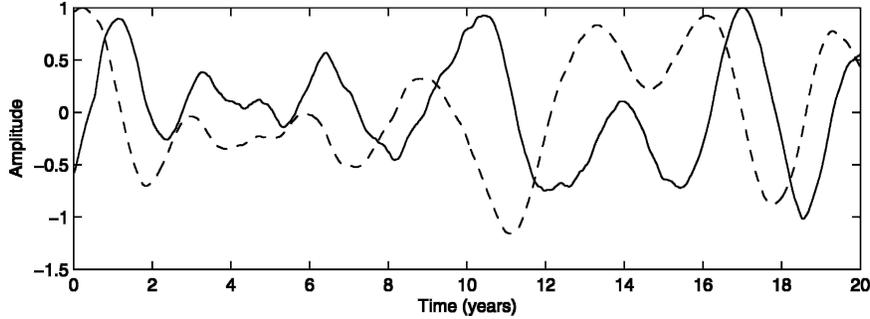


FIG. 3. Amplitudes of zonal wavenumber-2 SST and SLP anomalies (solid and dashed curves, respectively) under white-noise forcing of SST. Data have been low-pass filtered with a cutoff period of 1 yr, and amplitudes have been scaled.

ture in the latitude band  $40^{\circ}$ – $70^{\circ}$ S reveals a correlation between the two of 0.59, which is significant at the 99% level. That is, the linear relation (9) explains approximately 35% of total variability in monthly mean SLP over the Southern Ocean. This result is consistent with Karoly (1989), who found a dominant equivalent barotropic response at high southern latitudes.

If the vertical advection term in (8) is assumed to be dominated by the horizontal advection term, and  $v_a$  is related to  $p$  through geostrophy  $v_a = p_x / (f_0 \rho_a)$ , then the equation for  $p$  is

$$[\bar{U}_a + \gamma(f_0 \rho_a) \bar{T}_{ay}] p_x = -\lambda_a p + b \gamma \lambda_a T. \quad (10)$$

Now if zonally harmonic solutions are required for  $p$  of wavenumber  $k$ , then the following diagnostic equation relating  $p$  to  $T$  can be written

$$p = b \gamma \lambda_a / [(\bar{U}_a + \gamma \bar{T}_{ay} / f_0 \rho_a) i k + \lambda_a] T. \quad (11)$$

Thus, assuming that (9) explains all SLP variability, the phase of  $p$  relative to  $T$  is simply determined by the coefficient of  $T$  in (11). We use the following set of parameters as appropriate for the Southern Ocean-atmosphere system:  $f_0 = -1.19 \times 10^{-4} \text{ s}^{-1}$ ,  $\beta = 1.32 \times 10^{-11} \text{ s}^{-1} \text{ m}^{-1}$ ,  $\rho_a = 1.23 \text{ kg m}^{-3}$ ,  $\bar{U}_a = 10.0 \text{ m s}^{-1}$ ,  $\bar{T}_{ay} = 0.4^{\circ} \text{C} (\text{lat})^{-1}$ ,  $\lambda_a^{-1} = 2 \text{ weeks}$ ,  $b = 134$ , and  $\gamma = 200 \text{ Pa } (^{\circ} \text{C})^{-1}$ . These are identical to the values of these parameters used by Qiu and Jin (1997). For these parameter values, the phase difference between  $p$  and  $T$  as a function of  $k$  is approximately the arctangent of  $(6k \times 10^6)$ . Thus, under the assumption of an equivalent barotropic response to thermal forcing, SST anomalies with zonal wavenumbers 2 and 3 lag the SLP anomalies that they induce by approximately  $74^{\circ}$  and  $79^{\circ}$ , respectively, close to the  $90^{\circ}$  phase difference inferred from observations. Clearly the exact phase relationship is sensitive to estimated parameters such as  $\bar{U}_a$ ,  $\bar{T}_{ay}$  and  $\lambda_a$ . The phase difference approaches  $90^{\circ}$  as  $\lambda_a$  or  $\bar{T}_{ay}$  is decreased or as  $\bar{U}_a$  is increased. For example, for

$\lambda_a = (3 \text{ weeks})^{-1}$ ,  $\bar{T}_{ay} = 0.3^{\circ} \text{C} (\text{lat})^{-1}$ , and  $\bar{U}_a = 15.0 \text{ m s}^{-1}$ , SLP leads SST by  $84.9^{\circ}$  and  $86.6^{\circ}$  for wavenumbers 2 and 3, respectively.

The stochastic excitation of these coupled anomalies is demonstrated in Fig. 3. A zonal wavenumber-2 structure for both  $T$  and  $p$  is assumed, so that the system is composed of (11) and (3) with tangent linear operator as given in (5). Figure 3 shows the amplitude of the wavenumber-2 SST anomalies in the model when a white-noise forcing term is added to the right-hand side of (3) (solid curve), and also the resulting SLP anomalies (dashed curve). The stochastic noise excites SST oscillations predominantly at the eigenfrequency ( $kU = 3.64 \text{ yr}^{-1}$ ) corresponding to wavenumber 2, and it may be seen that the SLP anomalies tend to lead SST by close to 1 yr, approximately a quarter cycle and hence consistent with observations.

## 6. Conclusions

The ability of a simple stochastically forced advection equation to describe interannual Southern Ocean SST anomalies associated with the ACW has been investigated. It has been shown that for parameters that are reasonable for the Southern Ocean such a model can possess eigenmodes that resemble the SST signature of the ACW in amplitude, frequency, and zonal wavenumber. It has also been shown that when an eddy diffusivity is included, variance is naturally maintained in such a model at low zonal wavenumbers.

Upon extending the model to allow variability along the path of the ACC, the least damped of the system eigenmodes were found to be interannual and of low wavenumber. As a result of the decay rate increasing only gradually with wavenumber for small wavenumbers, a long stochastically forced simulation of the system demonstrated variance spread among low wavenumbers and interannual frequencies. Such a broad

spread of spectral power is not seen in the short record of ACW observations, however, similar time-length segments sampled from the model simulations can resemble the ACW SST spectra, as seen in Fig. 1a, suggesting the possibility that the ACW observations may be consistent with a short sampling from a broadband stochastically forced system. Anisotropic forcing of our model was able to sustain a dominant wavenumber/frequency through preferential excitement of one mode; excitation of the wavenumber-3 mode for instance was demonstrated by forcing the model either with a stochastically varying random phase wavenumber-3 pattern or with isotropic harmonic forcing near the mode's resonant frequency. Such a wavenumber-3 forcing could be derived from vertical or horizontal heat flux processes linked to the dominant wavenumber-3 pattern of Southern Hemisphere SLP. As discussed above, our stochastic forcing through cross-path (meridional) advection of mean SST can be regarded as an equivalent vertical heat flux. In either context, we have demonstrated that stochastic forcing of the mean SST field is of sufficient magnitude in the Southern Ocean to induce anomalous upper-ocean temperature fluctuations of order 1°C, of the same order or slightly higher than those seen in the ACW observations. Overestimation of the SST anomalies would be expected of the model, owing to the absence of nonlinearity, which would tend to curb anomaly growth. In contrast, the weaker-than-observed ACW signals seen in coarse resolution coupled GCM simulations may result in part from their inability to include the realistic levels of high-frequency forcing considered here.

Our results suggest that explicit coupling to a dynamic ocean or atmosphere may not be necessary in order to produce an ACW-like response in the system. This supports previous studies that have been able to understand much of the interannual Southern Ocean SST variability in coupled models using uncoupled advection equations similar to (3). In addition, it was shown that SLP anomalies driven through an equivalent barotropic response to ocean thermal forcing can reproduce a phase relationship between SLP and SST that resembles that associated with the ACW. Thus it is possible that the ACW, or at least some component of it, can be explained as a stochastically forced mode of the SST field that forces a correlated response in SLP, but without any significant feedback.

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