The Ocean Circulation in Thermohaline Coordinates

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The thermohaline streamfunction is presented. The thermohaline streamfunction is the integral of transport in temperature - salinity space and represents the net pathway of oceanic water parcels in that space. The thermohaline streamfunction is proposed as a diagnostic to understand the global oceanic circulation and its role in the global movement of heat and freshwater. The coordinate system used is fully Lagrangian. As such, physical pathways and ventilation timescales are naturally diagnosed, as are the roles of the mean flow and turbulent fluctuations. As potential density is a function of temperature and salinity, the framework is naturally isopycnal and is ideal for the diagnosis of water-mass transformations and advective diapycnal heat and freshwater transports. Crucially, the thermohaline streamfunction is computationally and practically trivial to implement as a diagnostic for ocean models. Here, the thermohaline streamfunction is computed using the output of an equilibrated intermediate complexity climate model. It describes a global cell, a warm tropical cell and a bottom water cell. The streamfunction computed from eddy-induced advection is equivalent in magnitude to that from the total advection, demonstrating the zero order importance of parameterised eddy fluxes in oceanic heat and freshwater transports. The global cell, being clockwise in thermohaline space, tends to advect both heat and salt towards denser (poleward) water-masses in symmetry with the atmosphere’s poleward transport of moisture. A re-projection of the global cell, from thermohaline to geographical coordinates reveals a thermohaline mean circulation reminiscent of the schematised ‘global conveyor’.
1. Introduction

It has long been recognised that the ocean displays variability across a large spectrum of spatial scales, from global scale oceanic gyres and circumpolar currents to millimetre scale turbulent motions, and on a myriad of temporal scales from millennial overturning to waves with micro second periods (Davis et al. 1981). All temporal and spatial scales of motion contribute to the global ocean circulation and its role in the climate system (Munk and Wunsch 1998). Despite this, descriptive oceanographers have identified specific pathways of net oceanic motion from observed tracers fields (Wüst 1935; Deacon 1937; Gordon 1986). These pathways have been schematised in a variety of ways, the ‘global conveyor’ of Broecker (1991) being the most universally recognised (see Richardson 2008, for a review of such schematics).

Over the past half century, numerical models have moved to finer and finer resolution and have progressively resolved an increasing number of scales. A key challenge remains however: distilling the plethora of motions of a numerical model into simplified indices and diagrams with minimal loss of information. It is pertinent, also, to test integral metrics of the ocean circulation and quantify how they relate to the climate system.

A common way of understanding the global circulation is to average oceanic velocities at constant latitude and depth and look at the circulation in a meridional-vertical coordinate. The circulation revealed is known as the Meridional Overturning Circulation (MOC, Kuhlbrodt et al. 2007). The MOC describes net vertical and meridional motion. it is often used to infer how the ocean distributes properties around the globe by transposing the diagnosed MOC onto zonally averaged quantities. This approach is somewhat valid in the North
Atlantic where a strong mean overturning exists and properties are reasonably homogeneous at constant depth and latitude. In other regions however, overturning cells in the depth-latitude plane do not correspond to property transports at all. In the Southern Ocean, for example, a vigorous ‘Deacon Cell’ exists, apparently driving 30-40 Sv of light surface water down to the depth of Drake Passage at around 2000m (Manabe et al. 1990). However, when the circulation is averaged in density-latitude space, correlations between the meridional velocity and the thickness of density layers counter the Deacon Cell (Dős and Webb 1994). Thus, by averaging the flow in density, rather than depth space, a complimentary view, more consistent with the distribution of tracers, is revealed.

In the example of the Southern Ocean, the vertical coordinate, density, is used to capture meridional exchanges of water-masses. Nycander et al. (2007) and Nurser and Lee (2004) propose diagnosing the ocean circulation in density - depth coordinates to capture the vertical exchanges of water-masses. Using this approach, Zika (2011) demonstrate that the processes which contribute to meridional exchanges of water-masses in the Southern Ocean can be very different to those which contribute to vertical exchanges. Boccaletti et al. (2005) and Ferrari and Ferreira (2011) have analysed the meridional circulation in temperature-latitude coordinates and reveal a different circulation to that in density-latitude coordinates. Observed tracer distributions reveal oceanic exchanges of both heat and salt which are not only meridional, but also zonal, between ocean basins, and vertical, between the ocean surface and the interior (e.g. Broecker 1991; Gordon 1991; Schmitz 1996). We seek a coordinate system which captures this net global circulation.

Principally, the MOC is considered important in the climate system because of its role in transporting heat, freshwater, nutrients and carbon. By transporting heat and freshwater
around the globe the ocean influences global temperatures and plays a key role in the hydro-
logical cycle. The water-masses through which the global-conveyor is thought to travel are
each defined by unique temperature and salinity properties. Here we investigate the ocean
circulation in temperature-salinity (thermohaline) coordinates. By doing so, we are able to
determine whether a ‘global conveyor’ of the scale previously proposed based on observed
tracer fields (Gordon 1986; Broecker 1991) exists in a climate model.

Zika and McDougall (2008); Zika et al. (2009b) use a conservative temperature-neutral
density coordinate (locally equivalent to a rotated conservative temperature-salinity coordi-
nate) to understand the balance of advection and mixing in the ocean interior from observ-
ations. They establish the fundamental link between flow in this coordinate and diabatic
processes, and have extended this to a general inverse method (Zika et al. 2009a, 2010).
Some previous studies have analysed numerical model output in a thermohaline coordinate
(Cuny et al. 2002; Marsh et al. 2005). Indeed, Blanke et al. (2006) compute a streamfunction
in thermohaline space for a regional model of the North Atlantic using Lagrangian particle
tracking. Here we compute a streamfunction in thermohaline coordinates from the output
of a global ocean model using a straight-forward methodology. We then discuss the meaning
and implications for flow in such a coordinate system.

Section 2 of this article defines the thermohaline streamfunction (THS) and describes how
it can be easily diagnosed from a finite volume ocean model. Section 3 describes the THS
as diagnosed from a climate model. Section 4 discusses the Eulerian mean and fluctuating
contributions to the THS and in Section 5 the calculation of advective diapycnal heat and
salt fluxes is discussed. Section 6 describes the diagnosis of a thermohaline transit time while
Section 7 shows a diagnosis of the THS for different basins and the use of the THS to infer
water-mass pathways. We summarise the main conclusions in Section 8.

2. The thermohaline streamfunction

Here we review some standard streamfunction diagnostics used in the literature, then describe the computation of a thermohaline streamfunction.

a. Barotropic and Meridional Overturning Streamfunctions

Take a flow in the 3 coordinates: longitude, $x$, latitude, $y$ and height, $z$ bounded by $[x_1, x_2]$, $[y_1, y_2]$ and $[-H(x, y), \eta(x, y)]$ respectively with velocity $\mathbf{u} = [u, v, w]$. If we have

$$\nabla \cdot \mathbf{u} = 0; \quad (1)$$

we can define a streamfunction, $\psi_{xy}$, such that

$$\frac{\partial \psi_{xy}}{\partial y} = -\int_{-H}^\eta u \, dz; \quad \frac{\partial \psi_{xy}}{\partial x} = \int_{-H}^\eta v \, dz. \quad (2)$$

So long as $v = 0$ at $y = y_1$ (in the case of the global ocean, $y_1$ would be a latitude circle wholly inside the Antarctic continent), the streamfunction is diagnosed using

$$\psi_{xy}(x, y) = -\int_{y_1}^y \int_{-H}^\eta u(x, y', z) \, dz \, dy'. \quad (3)$$

$\psi_{xy}$ is commonly known as the barotropic streamfunction. The barotropic streamfunction for an intermediate complexity climate model (to be discussed in detail in Section 3) is plotted in Fig. 1. The barotropic streamfunction reveals the wind driven basin scale gyres, their boundary currents, and the pathway and strength of the Antarctic Circumpolar Current.
Alternatively, integrating at constant depth and latitude, one can derive a meridional overturning streamfunction $\psi_{zy}$ such that

$$\frac{\partial \psi_{zy}}{\partial y} = -\int_{x_1}^{x_2} w \, dx ; \quad \frac{\partial \psi_{zy}}{\partial z} = \int_{x_1}^{x_2} v \, dx \quad (4)$$

and again, so long as $w = 0$ at $z = -H$ (e.g. below the deepest topography), $\psi_{zy}$ can be diagnosed as:

$$\psi_{zy}(y, z) = -\int_{-H}^{z} \int_{x_1}^{x_2} v(x, y, z') \, dx \, dz'. \quad (5)$$

The meridional overturning streamfunction (MOC) for an intermediate complexity climate model is plotted in Fig. 2. The MOC reveals meridional cells linking the equatorial and poleward waters and the surface and deep waters. As discussed in Section 1, the MOC does not accurately represent the meridional exchanges of waters of different densities and temperatures. In order to diagnose such exchanges, and infer the processes which contribute to those exchanges, it is illuminating to compute a streamfunction as a function of latitude and a second, time evolving tracer. Following Ferrari and Ferreira (2011), we define a meridional streamfunction $\psi_{Cy}$, for an arbitrary tracer $C$, such that

$$\psi_{Cy} = \int \int_{C' \leq C} v \, dx \, dz \quad (6)$$

where $\int \int_{C' \leq C} dx \, dz$ is the area over a surface of constant latitude, where $C' \leq C$. The streamfunction $\psi_{Cy}$ can be used to diagnose the advective meridional transport of $C$ and to understand which mechanism give rise to that transport (Ferrari and Ferreira 2011).

Equation (7) applied to an instantaneous velocity field gives an instantaneous streamfunction. In this case the streamfunction represents not only the rate at which water parcels move from one concentration, $C$, to another, but also the rate at which iso-surfaces of constant
move in space. Averaging over some period $\Delta t$ we may diagnose a mean streamfunction $\Psi_{Cy}$ such that

$$
\Psi_{Cy} = \frac{1}{\Delta t} \int_t^{t+\Delta t} \int_{C' \leq C} v \, dx \, dz \, dt
$$

(7)

Here and throughout, time mean streamfunctions will be defined using a capital $\Psi$. Eulerian mean and fluctuating components will later be distinguished using $\overline{\Psi}$ and $\Psi'$ respectively. As (Ferrari and Ferreira 2011) point out, if $C$ is steady over the period $\Delta t$, (i.e. $\int_t^{t+\Delta t} (dC/dt) \, dt = 0$), then $\Psi_{Cy}$ represents the flow of water from one $C$ value to another. Whether or not the flow is steady, $\Psi_{Cy}$, can still be diagnosed, in the unsteady case it represents both the transformation of water parcels and the movement of the tracer (Nurser and Marsh 1998).

Here we compute the $\Psi_{y\sigma_2}$, that is, a streamfunction in potential density - latitude coordinates ($\sigma_2$ being density referenced to 2000m depth; Fig. 2 b). The density-latitude streamfunction gives a complementary view of the circulation to the depth-latitude streamfunction (Fig. 2a). In particular the Deacon Cell is reduced to 3-4 Sv in the density-latitude case. The parameterised eddy-induced velocity and zonal asymmetries allow a circulation to exist in the Southern Ocean in latitude-depth space without large exchanges of mass (density) across latitude circles. In addition, the bottom water cell around Antarctica and the deep 'abyssal cell' below 2000m depth are here linked together in one bottom water cell at densities greater than 37 kg m$^{-3}$.

What is not clear from the depth-latitude, nor the density latitude streamfunction, is the route taken by dense waters after they have sunk in formation regions such as the North Atlantic. Both diagnostics show a sinking around 60°N. Where these waters eventually upwell
and which geographical regions they transit through (either isopycnally or diapycnally) is unclear.

b. A streamfunction in two general coordinates

We seek a streamfunction which is not solely meridional, but can also encapsulate vertical and zonal motions as well. As such, we compute a streamfunction $\psi_{C_1C_2}$ in terms of two tracers, $C_1$ and $C_2$. Hence

$$\psi_{C_1C_2} = \int_{C_1' \leq C_1 | C_2} \mathbf{u} \cdot \mathbf{n}_{C_2} \, dA$$

(8)

where $\mathbf{n}_{C_1}$ is the direction normal to a $C_2$ iso-surface and $\int_{C_1' \leq C_1 | C_2} \, dA$ is the area over the $C_2$ iso-surface where $C_1' \leq C_1$. Again, the instantaneous $\psi_{C_1C_2}$ represents both the transformation of water parcels from different tracer concentrations and the adiabatic movement of tracer iso-surfaces (see Griffies 2007, pages 138-141 for a discussion of dia-surface flow).

The mean streamfunction in the $C_1 - C_2$ coordinate is then

$$\Psi_{C_1C_2} = \frac{1}{\Delta t} \int_t^{t+\Delta t} \int_{C_1' \leq C_1 | C_2} \mathbf{u} \cdot \mathbf{n}_{C_1} \, dA \, dt$$

(9)

In this study we will only consider cases where tracers are statistically steady. Interpretations of unsteady streamfunctions are left to future work.

Note that a streamfunction $\Psi_{C_1C_2}$ could be formulated for iso-surfaces of constant $C_1$ where $\Psi_{C_1C_2} = -\Psi_{C_2C_1}$. Replacing $C_2$ with the meridional coordinate $y$, (9) is equivalent to (7) as $\mathbf{u} \cdot \mathbf{n}_y = v$. Indeed, one could also replace $C_1$ with the vertical coordinate $z$ and recover the mean of the latitude-depth overturning $\Psi_{yz}$ (5; i.e. the MOC). Hence (9) represents a general equation for a streamfunction in two coordinates, whether in standard geographical
coordinates or coordinates defined by time variable tracers.

For the remainder of this study we will consider oceanic flow in thermohaline coordinates. That is, where our two tracer variables are potential temperature, $\theta$, and salinity, $S$. The mean thermohaline streamfunction is given by

$$\Psi_{\theta S} = \frac{1}{\Delta t} \int_{t}^{t+\Delta t} \int_{\theta \leq \theta | S} \mathbf{u} \cdot \mathbf{n} \ dA \ dt. \quad (10)$$

In practice (9) could be applied to any three-dimensional, incompressible flow. We use $\theta$ and $S$ as these are the variables which describe heat and salt content respectively in our model. In calculations based on more precise models or observations, the appropriate heat variable is conservative temperature ($\Theta$; McDougall 2003) and the appropriate salt variable is absolute salinity ($S_A$; McDougall et al. 2010).

If the flow and tracer distributions are steady, $\Psi_{\theta S}(S, \theta)$ represents the transformation of water-masses from different temperature and salinity values; i.e. the warming and freshening of water parcels as they move through the ocean. This conversion need not be diapycnal as water parcels can have compensating changes in heat and salt and remain at the same isopycnal.

Schematically represented in Fig. 3 are four streamfunctions: two in fixed geographical coordinates, $x - y$ and $z - y$, one with a Lagrangian vertical coordinate, $\theta - y$ and finally the $\theta - S$ streamfunction in purely Lagrangian co-ordinates. The flow in Fig. 3 is reminiscent of both a mid-latitude gyre and diffusively upwelled thermohaline conveyor (Stommel and Arons 1960). A gyre circulation moves warm salty waters poleward (Fig. 3 a). The warm salty water then cools becoming cold and salty. The surface waters then freshen with some sinking into the deep ocean (Fig. 3 a). The surface gyre circulation warms and evaporates as
it moves southward becoming warm and salty again. The deep portion warms and freshens via interior mixing as it upwells (Fig. 3 b). Averaging at constant latitude we see that this circulation transports warm water northward and cold water southward (Fig. 3 c). Averaging in temperature salinity space we see that, in this special case, both the gyre circulation and the diffusively upwelled overturning involve a circulation from warm and fresh to warm and salty, then to cold and salty, to cold and fresh and back to warm and fresh.

Ocean models do not describe a continuous flow field and temperature and salinity distributions but rather, a discretised approximation of it. The most common discretisation is one where the equations of motion and property conservation are solved on a grid defined by finite volumes. For each volume, a property concentration is assigned, as are transports across grid volume interfaces.

Here we describe how $\Psi_{\theta S}$ is calculated from a finite volume ocean model. Consider a model with $N$ grid box interfaces and $M$ discrete time steps. Volume fluxes, $U_{ij}$, across all grid box interfaces, $i$, at time steps, $j$, are determined from the interface velocity and the interface area (e.g. $U = [u\Delta y\Delta z; v\Delta x\Delta z; w\Delta x\Delta y]$ for longitudinal, latitudinal and vertical grid spacing $\Delta x$, $\Delta y$ and $\Delta z$ respectively); and the respective temperatures at the interfaces $\theta_{ij}$ and the salinities of the adjoining grid boxes $S_{ij}^+$ and $S_{ij}^-$ are stored in computer memory. Then, over all time steps the thermohaline streamfunction is computed using
\[ \Psi_{\theta S}(S, \theta) = \frac{1}{M} \sum_{j=1}^{M} \sum_{j=1}^{N} \delta_{ij}^\theta \delta_{ij}^S U_{ij}, \]

\[ \delta_{ij}^\theta = \begin{cases} 
1 & \text{if } \theta_{ij} \leq \theta, \\
0 & \text{otherwise.} 
\end{cases} \quad (11) \]

\[ \delta_{ij}^S = \begin{cases} 
1 & \text{if } S_{ij}^+ > S \text{ and } S_{ij}^- < S, \\
-1 & \text{if } S_{ij}^+ < S \text{ and } S_{ij}^- > S, \\
0 & \text{otherwise.} 
\end{cases} \]

Above, the delta function \( \delta^\theta \) eliminates from the calculation all interfaces where \( \theta_i > \theta \) and \( \delta^S \) eliminates fluxes which do not cross the \( S \) iso-surface and also sets the sign of the flux depending on whether the flow is up gradient or down gradient.

Fig. 4 shows how a thermohaline streamfunction, \( \Psi_{\theta S} \), is calculated from a discretised field. In Fig. 4, the grid boxes where \( \delta^S \delta^\theta = 1 \) or -1 are those along the light blue line in panel c) between \( S' < S \) and \( S' > S \) with \( \theta' < \theta \). Computing \( \Psi_{\theta S} \) for a velocity field such as the output of an ocean model, we effectively integrate over all water-masses of like properties in time and space.

3. The thermohaline streamfunction of a climate model

We diagnose the ocean component of the final 10 years of a 3000 year simulation of the University of Victoria Climate Model (1.8° latitude by 3.6° longitude grid spacing, 19 levels, 2D energy balance atmosphere) as used by (Sijp et al. 2006, specifically their GM case). The ocean model is MOM2 (GFDL MOM Version 2.2, Pacanowski 1995). The
vertical mixing coefficient increases with depth, taking a value of $0.6 \times 10^{-4}$ m$^2$ s$^{-1}$ at the surface and increasing to $1.6 \times 10^{-4}$ m$^2$ s$^{-1}$ at the bottom. The model employs the eddy-induced advection parameterization of Gent and McWilliams (1990) with a constant diffusion coefficient of 1000 m$^2$ s$^{-1}$. Tracers are diffused in the isopycnal direction with a constant coefficient of 2000 m$^2$ s$^{-1}$.

The ocean model displays a plausible global $\theta - S$ distribution and MOC and by the year 3000 the ocean model has reached a stable equilibrium. The model is typical of those used to understand different climate states and the stability of the global overturning circulation. As per all such ocean climate models, the model MOC displays several isolated cells including a shallow clockwise (north) and an anti-clockwise (south) cell in the tropics, a Northern Hemisphere deep cell, a bottom water cell towards Antarctica, a Deacon Cell between 60°S and 40°S and finally an abyssal cell below 2000m depth.

A scatter plot of ocean $\theta - S$ with a colour code corresponding to ocean basin and latitude allows one to relate points in $\theta - S$ space to particular geographical regions (Fig. 5). The warmest and saltiest waters are found in the North Atlantic (red), as are colder salty waters reaching below 5°C (red). The coldest and freshest waters are found in the Arctic (red), cold and modestly fresh waters are found in the Southern Ocean (yellow) and fresh but generally slightly warmer waters in the North Pacific (dark blue). The warmest waters are found in the equatorial Indo-Pacific Oceans (cyan). Only along a locus of water-mass classes, narrow in the saline coordinate, do waters from the three major ocean basins exist together.

Using monthly velocity, potential temperature and salinity fields from the ocean model, we compute the mean thermohaline streamfunction (Fig. 6). It reveals, as its dominant feature, a large15 Sv cell. This global cell reaches from the warm salty waters found only in the
Atlantic, through to the cold and fresh waters found largely in the Southern Ocean and then
to the warm fresh waters found only in the Indo-Pacific basin. Two other counterclockwise
cells appear in the THS. These are the very warm tropical cell ($\theta > 25^\circ C$) with a transport
of around 8 Sv and the very cold ($\theta < 2^\circ C$) bottom water cell, also with a transport of
around 8 Sv. Within the global cell lies a warm and a cold subcell, separated around the
$10^\circ C$ isotherm each with an additional transport of 5–10 Sv.

North Atlantic Deep Water (NADW) is characterised in this model as having a potential
temperature colder than $5^\circ C$ and a salinity of around 35 g / kg. Only 4 – 6 Sv achieves such
temperatures and salinities. NADW, at its coldest, is close to $2^\circ C$ with a salinity of around
34.5 g / kg. Once formed, the NADW warms, presumably through entrainment and interior
mixing. The NADW branch then joins the remaining streamlines heading towards colder
fresher regions of the ocean. In Section 6 we decompose this circulation by ocean basin.

A circulation of around 6 Sv links water-masses found uniquely in the Atlantic, Southern
and Indo-Pacific (compare Fig. 5 and Fig. 6) The many vertical and horizontal circulations,
one integrated in $\theta - S$ space, give rise to one interconnected flow. In effect, this is a

4. Eddy-induced and seasonal transports

In the ocean model discussed here, as in virtually all contemporary ocean models which
do not explicitly resolve eddy fluxes, tracers are advected by both an Eulerian mean velocity
and an eddy-induced velocity \( \mathbf{u}^{GM} \) such that

\[
\mathbf{u} = \mathbf{u}^{EM} + \mathbf{u}^{GM}.
\] (12)

The eddy-induced velocity is prescribed using the scheme of Gent and McWilliams (1990).

In the MOC, \( \mathbf{u}^{GM} \) is significant mainly in the Southern Ocean and North Atlantic where isopycnal slopes are very large. When computing the THS of Fig. 6, we used both the eddy-induced and Eulerian velocities and averaged these for each monthly field. We can now assess the influence of the Eulerian flow and the eddy-induced flow and the fluctuations actually resolved by the model by making the following decomposition:

\[
\Psi^{EM}_{\theta S} (S, \theta) = \int_{\theta' \leq \theta_{\|}} \mathbf{u}^{EM} \cdot \mathbf{n} \, dA \, dt,
\]

\[
\Psi^{GM}_{\theta S} (S, \theta) = \int_{\theta' \leq \theta_{\|}} \mathbf{u}^{GM} \cdot \mathbf{n} \, dA \, dt
\] (13)

where the overline represents an Eulerian average (i.e. \( \bar{\xi} = \frac{1}{\Delta t} \int_{t}^{t+\Delta t} \xi \, dt \) for some variable \( \xi \)).

At its peak, the Eulerian mean contribution to the thermohaline streamfunction, \( \Psi^{EM}_{\theta S} \), is 39 Sv (Fig. 7b). The global cell of the Eulerian plus eddy-induced THS is 15 Sv and peaks at 25 Sv in its sub-cells (Fig. 6). The Eulerian mean THS is compensated by an anti-clockwise circulation due to the eddy-induced velocity of around 5 - 10 Sv (Fig. 7b).

This compensation occurs across a significant fraction of the water-mass classes covered by the global THS cell (-1°C > \( \theta \) > 15°C and 34 g / kg > \( S \) > 35 g / kg). We find that, while \( \mathbf{u}^{GM} \) is small across most latitudes, it is large where water flows from one water-mass to another, particularly in the cooler fresher water-masses associated with the Southern Ocean.

As above, the eddy-induced velocity is introduced to account for the effect of transient eddies not resolved on the 1.8° by 3.6° grid. The model does however, resolve fluctuations on
seasonal and inter-annual timescales and hence we do expect a difference between $\Psi_{\theta S}$ and $\Psi_{\theta S} = \Psi_{\theta S}^{EM} + \Psi_{\theta S}^{GM}$; the difference being the role played by resolved temporal fluctuations ($\Psi_{\theta S}'$, Fig. 7d). The major difference between the two is in the tropical waters ($\theta > 20^\circ C$) and in the region linking the warm salty portion and the cold fresh portion of the global cell ($\theta \approx 15^\circ C$ and $S \approx 35 g/kg$). The ocean model used here is known to display a robust seasonal cycle but small interannual fluctuations. It is thus likely that that in this model, and perhaps in the real ocean, seasonal fluctuations that contribute around 5 Sv of exchange between warm salty and cold fresh waters.

5. Diapycnal fluxes and the symmetric diapycnal flux theorem

By way of introduction to the idea of diagnosing the advective heat transport from a streamfunction, we again revisit Ferrari and Ferreira (2011) where their latitude-$\theta$ streamfunction $\Psi_{y-\theta}$ can be used to diagnose the advective meridional heat transport (MHT) using

$$MHT(y, \theta) = \int_{-\infty}^{\infty} \rho_0 c_p \Psi_{y\theta} d\theta$$  \quad (14)

where $\rho_0$ is a reference density and $c_p$ is the heat capacity of sea water. Equation (14) represents the flux of heat in the meridional direction due to advection, at the latitude $y$. In the case of the thermohaline streamfunction, $\Psi_{\theta S}$ can be used to determine a heat function describing the flux of heat across isohalines and a salt function describing the flux of salt across isotherms.

As potential density is a function of temperature and salinity, a heat function and salt
function can also be defined, describing the flux of heat and salt across isopycnals. The total
heat and salt transports across an isopycnal are given by

\[
F_{\text{Heat}}(\sigma) = \rho_0 c_p \int_{-\infty}^{\infty} \frac{\partial \Psi_{\theta S}}{\partial \theta} \theta \, d\theta|_\sigma + \rho_0 c_p \int_{-\infty}^{\infty} \frac{\partial \Psi_{\theta S}}{\partial S} \theta \, dS|_\sigma, \tag{15}
\]

\[
F_{\text{Salt}}(\sigma) = \int_{-\infty}^{\infty} \frac{\partial \Psi_{\theta S}}{\partial \theta} S \, d\theta|_\sigma + \int_{-\infty}^{\infty} \frac{\partial \Psi_{\theta S}}{\partial S} S \, dS|_\sigma. \tag{16}
\]

The salt transport can be converted into the more tangible ‘freshwater transport’, \(F_{\text{FW}}\),
in units of Sverdrups simply using \(F_{\text{FW}} = -F_{\text{Salt}}/\overline{S}\). Here we use \(\overline{S} = 35 \, \text{g} / \text{kg}\). The
diapycnal heat and freshwater transports across potential density surfaces \((\sigma_0)\) from the
ocean model simulation are shown in Fig. 8.

For most isopycnal ranges, the ocean transports heat towards denser waters and freshwater
towards lighter waters. This is consistent with a balance between the ocean circulation
and the atmosphere; the subtropical waters being associated with a heat flux into the ocean
and a freshwater flux out of the ocean. The denser polar waters are associated with a heat
flux out of the ocean and a freshwater flux in. Greatbatch and Zhai (1990) used observed
meridional heat flux estimates and the ocean hydrography to estimate a global heat-function
in terms of a diffusivity for temperature. Our analysis suggests that accurate estimates of
atmosphere-ocean heat and freshwater fluxes could be used to estimate the thermohaline
streamfunction in a similar way. This would effectively be a combination of the surface
water-mass analysis method of Walin (1982) extended to 2 tracers and the tracer-contour
framework of Zika et al. (2010) used to relate interior diabatic pathways to mixing.

The sign of the diapycnal heat and salt flux is evident geometrically from the thermohaline
streamfunction itself. Clockwise cells advect heat and salt toward denser waters and
anticlockwise cells advect heat and salt toward lighter waters (Fig. 9). This symmetry
between downward heat and freshwater fluxes can be formally stated in the following.
For most $\theta$, $S$ and pressure ranges of the ocean, density increases with increasing salinity
and reducing temperature, i.e.

$$d\sigma = -\alpha \partial \theta + \beta \partial S$$  \hfill (17)

where $\alpha$ is a positive thermal expansion coefficient and $\beta$ is a positive haline contraction
coefficient. Here we will make the sole assumption that $\alpha/\beta$ is constant for each $\sigma$. Applying
the chain rule to (15) we obtain

$$F_{\text{Heat}}(\sigma) = \rho_0 c_p \int_{-\infty}^{\infty} \Psi_{\partial S \partial \theta} d\theta|_\sigma - \rho_0 c_p \int_{-\infty}^{\infty} \Psi_{\partial S \partial \theta} dS|_\sigma.$$  \hfill (18)

As $\Psi_{\partial S \partial \theta} = 0$ outside the maximum and minimum salinities and temperatures

$$F_{\text{Heat}}(\sigma) = -\rho_0 c_p \int_{-\infty}^{\infty} \Psi_{\partial S \partial \theta} d\theta|_\sigma - \rho_0 c_p \int_{-\infty}^{\infty} \Psi_{\partial S \partial \theta} dS|_\sigma.$$  \hfill (19)

and likewise for the salt flux

$$F_{\text{Salt}}(\sigma) = -\int_{-\infty}^{\infty} \Psi_{\partial S \partial \theta} dS|_\sigma - \int_{-\infty}^{\infty} \Psi_{\partial S \partial \theta} d\theta|_\sigma.$$  \hfill (20)

Now using (17; where $d\sigma = 0$) we find

$$F_{\text{Heat}}(\sigma) = -\rho_0 c_p \int_{-\infty}^{\infty} \Psi_{\partial S \partial \theta} d\theta|_\sigma - \rho_0 c_p \int_{-\infty}^{\infty} \Psi_{\partial S \partial \theta} \frac{\beta}{\alpha} dS|_\sigma.$$  \hfill (21)

$$F_{\text{Salt}}(\sigma) = -\int_{-\infty}^{\infty} \Psi_{\partial S \partial \theta} d\theta|_\sigma - \int_{-\infty}^{\infty} \Psi_{\partial S \partial \theta} \frac{\alpha}{\beta} dS|_\sigma.$$  \hfill (22)

and hence given we are assuming $\alpha/\beta$ is constant for constant $\sigma$ the ratio of diapycnal
advective heat flux to salt flux is simply

$$\frac{F_{\text{Heat}}(\sigma)}{F_{\text{Salt}}(\sigma)} = \rho_0 c_p \frac{\beta}{\alpha}.$$  \hfill (23)
The significance of (23), hereafter the diapycnal flux symmetry theorem, is that where there is a net advective heat flux across isopycnals by the statistically steady flow, salinity must be advected also. Hence freshwater must be advected in the opposite direction. So where the ocean advects heat toward denser water-masses, it must advect freshwater towards lighter water-masses.

In an ongoing study, we are investigating whether the diapycnal flux symmetry theorem has broader implications for global heat and freshwater transports. It may be the case that this theorem places constraints on the relationship between atmospheric moisture transport and ocean heat transport.

6. Thermohaline transit time

Given the thermohaline streamfunction and estimates of the mean gradients of temperature and salinity, it is possible to gain a measure of the time taken for a water parcel to make a full circuit in $\theta - S$ space. We name this measure the thermohaline transit time, $\mathcal{T}_{\theta S}(\Psi)$. As we discuss in the next Section, individual water parcels, even in the absence of dispersion, do not complete entire ‘loops’ in $\theta - S$ space, as with any three dimensional flow. However, the concept of transit time is still insightful as it gives a measure of the minimum time for a water parcel to be advected in this fully Lagrangian co-ordinate system.

For a fluid parcel following streamline in $\theta - S$ space, the time increment, $dt|_{\Psi}$, for a given $\theta$ and $S$ increments $\partial \theta|_{\Psi}$ and $\partial S|_{\Psi}$ is

$$
 dt|_{\Psi} = \frac{Dt}{D\theta} \partial \theta|_{\Psi} + \frac{Dt}{DS} \partial S|_{\Psi}.
$$

(24)
That is, the time taken for a fluid parcel to get between two points along a streamline separated by \( \partial \theta |_\psi \) and \( \partial S |_\psi \), depends on both the rate of change of temperature following the fluid parcel \( D\theta /Dt \), and the rate of change of salinity following the fluid parcel \( DS /Dt \).

If \( \theta \) and \( S \) are statistically steady, in order for a fluid parcel to change its temperature and salinity, it must be advected to somewhere with a different temperature and salinity such that

\[
\frac{D\theta}{Dt} = u \cdot \nabla \theta ; \quad \frac{DS}{Dt} = u \cdot \nabla S. \tag{25}
\]

By computing \( \Psi_{\theta S} \) from (10) we have naturally computed an average of both components of the velocity needed in (25) as (10) can be written

\[
\Delta \Psi|_S = \frac{1}{\Delta t} \int_t^{t+\Delta t} \int_{\theta}^{\theta+\Delta \theta} \frac{u \cdot \nabla \theta}{|\nabla \theta|} dA_S; \tag{26}
\]

\[
\Delta \Psi|_\theta = \frac{1}{\Delta t} \int_t^{t+\Delta t} \int_{S}^{S+\Delta S} \frac{u \cdot \nabla S}{|\nabla S|} dA_\theta. \tag{27}
\]

Assuming the absolute gradients \( |\nabla \theta| \) and \( |\nabla S| \) do not vary greatly in time or across the areas \( dA_S \) and \( dA_\theta \) respectively, we can make the following approximation:

\[
\frac{\Delta \Psi|_S}{\Delta A_S} = \left\langle \frac{u \cdot \nabla \theta}{|\nabla \theta|} \right\rangle \approx \frac{\left\langle u \cdot \nabla \theta \right\rangle}{|\nabla \theta|}; \tag{28}
\]

\[
\frac{\Delta \Psi|_\theta}{\Delta A_\theta} = \left\langle \frac{u \cdot \nabla S}{|\nabla S|} \right\rangle \approx \frac{\left\langle u \cdot \nabla S \right\rangle}{|\nabla S|}. \tag{29}
\]

Above, the angular brackets \( \langle \rangle \) represent the average in time and over an isotherm between \( S \) and \( S + \Delta S \) and the average over an isohaline between \( \theta \) and \( \theta + \Delta \theta \) respectively. From the time mean \( \theta \) and \( S \) fields of the model, we can compute \( \left\langle |\nabla \theta| \right\rangle \) and \( \left\langle |\nabla S| \right\rangle \) simply by averaging the gradients across the areas \( dA_S \) and \( dA_\theta \). Thus from (25) we have

\[
\frac{D\theta}{Dt} \approx \frac{\Delta \Psi|_S}{\Delta A_S} \left\langle |\nabla \theta| \right\rangle; \tag{30}
\]

\[
\frac{DS}{Dt} \approx \frac{\Delta \Psi|_\theta}{\Delta A_\theta} \left\langle |\nabla S| \right\rangle .
\]
Substituting into (24) then yields

\[ \frac{1}{\Delta \psi |_{\Delta A_{\psi}}} < |\nabla \psi| > \partial \theta |_{\psi} + \frac{1}{\Delta \psi |_{\Delta A_{\psi}}} < |\nabla S| > \partial S |_{\psi}. \]  

(31)

One can then estimate the time rate of change following the $\Psi_{\theta S}$ streamline by integrating (31) such that the total thermohaline transit time, $T_{\theta S}(\Psi)$, is

\[ T_{\theta S}(\Psi) = \oint dt |_{\psi} \approx \oint \frac{1}{\Delta \psi |_{\Delta A_{\psi}}} < |\nabla \theta| > \partial \theta |_{\psi} + \oint \frac{1}{\Delta \psi |_{\Delta A_{\psi}}} < |\nabla S| > \partial S |_{\psi}. \]  

(32)

We calculate $T_{\theta S}$ by following the longest -4 Sv streamline in $\theta - S$ space (Fig. 10). The full circuit takes 1,350 years using (32). The cumulative time taken along the contour shows how the fluid moves from the warmest freshest waters around 25° C and 34.4 g / kg to the cool high saline waters at 10° C and 35.5 g / kg in a relatively short time-scale of 100 years or so. This short time-scale is likely to be because of the strong, near surface wind driven circulation allowing rapid transitions between different water-masses. The transit from this water-mass (NADW) to the cold fresh waters around 0° C and 34.5 g / kg, a relative short distance in $\theta - S$ space, takes the considerable majority of the remaining 1250 year transit. These dense water-masses are found only in the deep ocean where advection is weak and temperature and salinity gradients are large meaning water-mass transitions occur gradually.

The thermohaline transit time of THS contours are shown in Fig. 11. In each case we take the longest streamline for a given transport value. The tropical cell, defined as anti-clockwise cells warmer than 10°C, has the shortest transit times of order 20 years. For the global cell and it’s subcells, the transit times range from hundreds of years and asymptote
to around 1500 years for the -2 Sv contour. We do not discern between subcells in the warmer or colder portions of the global cell. The longest bottom water cell transit times are equivalent to global cell at order 1500 years. Increased resolution of the THS in $\theta - S$ space may change the bottom water transit times estimated here.

7. Basin specific flow and water-mass pathways

Here we address whether streamlines in thermohaline space correspond to pathways of individual water parcels or whether this circulation is simply the accumulation of many, geographically independent cells. To answer this question we start by investigating the nature of the THS in separate geographical regions where such a separation is possible.

The North Atlantic is generally saltier than the Indian and Pacific basins. However the two share waters of like $\theta - S$ properties. Looking at the maximum temperatures and salinities along the sections which separate the Atlantic from the Indo-Pacific, we can determine $\theta - S$ ranges which are either ‘wholly’ within the Atlantic or ‘wholly’ within the Indo-Pacific, and of course, not in the Southern Ocean. That is, although like $\theta - S$ properties may exist in both basins, they are not geographically linked at any time. Any volume transport from Atlantic water to Indo-Pacific water must occur through some other intermediary $\theta - S$ range.

The $\theta - S$ ranges we choose for this strictly ‘non Southern Ocean’ water are $S > 35 g/kg$, $S > 34.8 g/kg$ and $\theta > 21.5^\circ C$ or $S < 34.8 g/kg$ and $\theta > 16^\circ C$ (i.e. only those regions in $\theta - S$ space which have green or red contours in Fig. 12). Any water in that range is either in the Atlantic, or in the Indo-Pacific, but not in the Southern Ocean. Hence, a streamfunction can
be defined independently by integrating form $\theta = \infty$ back to the edge of this domain in $\theta - S$ space. The result and the geographical domain of the two water-masses is shown in Fig. 12. The streamlines for the Indo-Pacific are shown in green and the Atlantic in red. Clearly the wide, anvil shape of the global THS is the result of a combination of a fresher Indo-Pacific branch and a saltier Atlantic branch. The warmest saltiest waters only exist in the Atlantic and the coldest freshest waters only exist in the the Indo-Pacific. The Indo-Pacific takes freshwater from the deep or southern waters and returns them as saltier waters while the Atlantic takes intermediate salinity waters and returns them as high salinity waters.

The same decomposition as is done for the warmest waters by ocean basin can be done for the coldest freshest waters by hemisphere. Cold fresh water exists in both the Southern Ocean and the Arctic/North Pacific. However, no cold fresh water exists around the Equator. Hence we can partition the two. Specifically, we compute a streamfunction only for waters with $\theta < 12^\circ C$ and $S < 34.5 g/kg$ for each hemisphere. The resulting Southern Ocean streamfunction is almost the same as that derived from the sum of both the Northern and Southern Hemispheres, demonstrating the dominant role of the Southern Ocean in the circulation in this $\theta - S$ range.

An interpretation of the global water-mass pathway thus emerges of water moving from the Atlantic to the Southern Ocean and into the Pacific. We now attempt to track this pathway more precisely by following a streamline from one basin to the next. We take the THS plotted in Fig. 12 and follow the $\theta - S$ values corresponding to the longest -4 Sv streamline. We start with the freshest point in the Atlantic on this contour. This contour is tracked until the edge of the Atlantic water-mass is reached at $S = 35 g/kg$. At this point the Atlantic streamfunction matches the global one. Continuing along the global streamline to
\( S = 34.5g/kg \) the global THS matches almost perfectly with the Southern Ocean and again to \( \theta = 12^\circ C \) where it matches with the Pacific. Although one could continue indefinitely, we stop where this streamline reaches \( \theta = 21.5^\circ C \) and \( S > 34.5g/kg \). By allowing streamlines to transit from one basin to the next, the circulation is no longer closed and no one cell exists in a unique sense.

In order to determine the possible geographical route of water parcels along this thermohaline pathway we colour water-masses in the ocean according to their location in \( \theta - S \) space. For each point along the contour, a colour is chosen corresponding to the ‘distance’ along the contour; distance being in \( \theta - S \) space, not geographical space. The colour is then used to label the actual points in the ocean which have that temperature and salinity. The resulting analysis is shown in Fig. 13. Following the pathway as it moves from the surface Atlantic to the North Atlantic, then down into the deep ocean, and eventually up into the Southern Ocean and out into the North Pacific, reveals a pathway reminiscent of the schematic diagrams of the thermohaline circulation, first depicted by Gordon (1986).

An important difference between the conveyor of Gordon (1986) and Broecker (1991) and that revealed by Fig. 13 is that the model diagnosed THS transits through the surface waters of the Southern Ocean with significant transformation of \( \theta - S \) properties. This is despite the meridional overturning in density space showing North Atlantic Deep Water upwelling diapycnally at mid-latitudes (Fig. 2). This suggests that waters are exchanged meridionally at constant density, but enter and exit with different temperatures and salinities (i.e. different spiciness).
8. Summary and conclusions

The circulation of a global ocean model has been presented in temperature salinity coordinates for the first time. The resulting thermohaline streamfunction, the THS, distils the ocean circulation into three distinct cells: a tropical warm cell, a global cell and an Antarctic Bottom Water cell. Many of the streamlines (around 4-5 Sv) of the global cell link the cold and salty waters of the North Atlantic to the warm fresh waters of the Indo-Pacific Ocean, providing a quantification of the models ‘global conveyor’. In addition, a number of novel advantages of computing a thermohaline streamfunction are revealed.

First, the THS can be used to diagnose the role of resolved and parameterised transient fluxes in water-mass transformation zones. Parameterised transient eddy fluxes are found to be leading order across the majority of water-mass classes. Averaging in thermohaline space provides a unique Lagrangian framework in which the role of diabatic processes and parameterised fluxes are highlighted.

Second, once the circulation is averaged in thermohaline coordinates, diapycnal heat and salt transports are easily quantified. Clockwise cells in thermohaline space flux heat and salt towards denser waters and anti-clockwise cells flux heat and salt towards lighter waters. This symmetry between diapycnal heat and salt fluxes is formalised with the symmetric diapycnal flux theorem (23). In our model, heat and salt are transported from light waters to dense waters by the clockwise global cell. Heat and salt are transported from light waters associated with sub-tropical latitudes to denser waters associated with polar regions. This poleward salt transport is in symmetry with the atmosphere’s poleward transport of freshwater at subtropical to subpolar latitudes.
A measure of the transit time of the global conveyor, the thermohaline transit time, is easily quantified from the THS. The thermohaline transit time in our model is 1350 years for typical streamlines associated with the conveyor. Such a simple diagnostic may prove useful in comparing the ventilation time of various models, especially in cases where the incorporation of more involved tracer diagnostics is prohibitive.

Separating the THS by region, the Southern Ocean is found to dominate the coolest and freshest water-masses. The THS trajectory from cool fresh waters to cool and salty waters is the sum of both a fresher tropical/subtropical branch in the Indian and Pacific Oceans and a saltier tropical to subpolar branch in the Atlantic Ocean. The sum of these three branches reveals a globally interconnected circulation.

A major difference between the global conveyor of this model and the schematic view of Gordon (1986) and Broecker (1991) is that most of the diagnosed THS (-5 to -15 Sv; narrow core of the THS in Fig. 6) occurs at the surface rather than in deep waters. In addition almost all streamlines (0 to -15 Sv) transit through the Southern Ocean. This is despite the majority of diapycnal upwelling in the model occurring in the abyssal ocean (Fig. 2). This implies that even a density space overturning, driven by interior diapycnal mixing, may still transit through the Southern Ocean in order for global balances of heat and salt to be maintained.

9. Acknowledgements

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REFERENCES


1 Mean barotropic streamfunction ($\Psi_{xy};$ Sv) from the ocean component of the final 10 year average of a 3000 year run of a climate model (described in Section 3). The 20 Sv (blue) and -20 Sv (red) contours are shown. $\Psi_{xy}$ is computed by integrating northward from the Antarctic, the value at the African continent is then subtracted.

2 Top: Mean depth-latitude streamfunction or MOC ($\Psi_{zy};$ Sv) from ocean component of a climate model (described in Section 3). Bottom: Density-latitude streamfunction (Sv). The 4 Sv (red) and -4 Sv (blue) streamlines are shown with solid contours.

3 Schematic describing four streamfunction diagnostics: a) barotropic streamfunction showing horizontal motion, b) meridional overturning streamfunction showing vertical motion, c) temperature-latitude streamfunction showing net effect of horizontal motion in transporting heat meridionally and d) thermohaline streamfunction showing net effect of all motion in transporting water from regions of different temperatures and salinities. Overlaid in a), b) and d) are schematic isolines of constant potential temperature and salinity (at the surface in a) and zonally averaged in b).
Schematic describing the way the thermohaline streamfunction is computed from the output of a finite volume ocean model. Model grid boxes are defined on some native coordinate (e.g. x and y, or y and z) by their salinity (a) and potential temperature (b). Where potential temperature $\theta' < \theta$ all fluxes are summed across grid box interfaces from salinity $S' < S$ to $S' > S$ in the native coordinate (c). Summing these fluxes, a thermohaline streamfunction at $(\theta, S')$ is determined and represented in thermohaline coordinates (d).

Scatter plot of potential temperature and salinity from the mean state of the ocean model. Each grid box is represented by one dot in $\theta - S$ space. The colour of each dot corresponds to its latitude and basin using the colour code shown in the insert.

The thermohaline streamfunction derived from 10 years of monthly, $u$, $v$, $w$, $\theta$ and $S$ fields of the ocean model. Positive values are anti-clockwise and negative values are clockwise. The 4 Sv streamline is shown in red and the -4 Sv streamline in blue.

Top left: THS from the sum of the time mean Euleran velocity and the time mean eddy-induced velocity ($\overline{u^{EM}} + \overline{u^{GM}}$). Top right: THS from the time mean Euleran velocity. Bottom left: THS from the time mean eddy-induced velocity. Bottom right: Difference between total THS and THS from the sum of the time mean Eulerian and eddy-induced velocities. That is, the contribution to the THS due to resolved fluctuations such as the seasonal cycle.
a) Thermohaline Streamfunction (THS, $\Psi_{\theta S}$) as in Fig 6 but with contours of surface referenced potential density ($\sigma_0$) overlaid. b) Diapycnal heat and c) diapycnal freshwater flux from the ocean model computed directly from the THS. The freshwater flux is simply computed as $F_{salt}/35g/kg$. The total (solid), Eulerian (dashed) and eddy-induced (dotted) components are shown. Consistent with the symmetric diapycnal flux theorem (23), a positive heat flux corresponds to a negative freshwater flux throughout.

Schematic describing the geometric relationship between the THS and diapycnal heat and salt fluxes. If water is advected toward denser waters on the warmer-saltier parts of an isopycnal surface ($\sigma$; left) this appears as a clockwise circulation in $\theta - S$ space (right). As the same trajectory has different temperatures as it crosses the same isopycnal surface it represents a net positive diapycnal transport of heat and salt. The circulation shown in physical space (left) can be both a horizontal (e.g. gyre in x-y space) circulation where this diapycnal transport is balanced by surface heating, cooling and evaporation minus precipitation and/or a vertical overturning (in z-y space) where the transport is balanced by interior mixing or nonlinear processes.

The accumulated thermohaline transit of the -4Sv streamline. The colour of each dot represents the relative time taken along the streamline estimated using only the thermohaline streamfunction itself and the mean $\theta - S$ gradients.
11 The thermohaline transit time of the longest THS streamlines in $\theta - S$ space (i.e. the longest streamlines in Fig. 6 for each transport). Shown are transit times for the global cell (green), the bottom water cell (temperatures below 10°C), and the tropical cell contours (longest positive streamlines with temperatures above 10°C). The Antarctic Bottom Water transit times may be underestimated due to interpolation onto a coarse $\theta - S$ grid.

12 The thermohaline streamfunction (THS) for various oceanic regions. Shown are: THS for water-masses found only in the Indian and Pacific basins (green; green volume in insert), water-masses found only in the Atlantic (red) and finally water-masses found only in the Southern Ocean (blue).

13 A thermohaline pathway projected into physical space. The -4 Sv THS contour is followed from the Atlantic water-masses (red in Fig. 12) to the Southern Ocean (blue in Fig. 12) and then finally to the Indo-Pacific water-masses (bottom right panel). At points equally spaced in $\theta - S$ space along the contour, a colour is chosen to represent water-masses with those water-mass properties (bottom right panel). All grid boxes in the corresponding region with potential temperatures and salinities within 0.1°C and 0.05 g / kg respectively of that point in $\theta - S$ space are then assigned the colour of that point. Top panel shows all grid boxes above 300m. Middle panel: all grid boxes below 300m. Bottom left panel: only Atlantic grid boxes. Bottom middle panel: Indo-Pacific grid boxes.
Fig. 1. Mean barotropic streamfunction ($\Psi_{xy}$; Sv) from the ocean component of the final 10 year average of a 3000 year run of a climate model (described in Section 3). The 20 Sv (blue) and -20 Sv (red) contours are shown. $\Psi_{xy}$ is computed by integrating northward from the Antarctic, the value at the African continent is then subtracted.
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