A dynamic, embedded Lagrangian model for ocean climate models, Part II: Idealised overflow tests

Michael L. Bates\textsuperscript{a,c,*}, Stephen M. Griffies\textsuperscript{b}, Matthew H. England\textsuperscript{a}

\textsuperscript{a}Climate Change Research Centre, University of New South Wales, Sydney, New South Wales, Australia
\textsuperscript{b}NOAA/Geophysical Fluid Dynamics Laboratory, 201 Forrestal Road, Princeton, NJ 08542, USA
\textsuperscript{c}Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

\textbf{Abstract}

Dense gravity current overflows occur in several regions throughout the world and are an important process in the meridional overturning circulation. Overflows are poorly represented in coarse resolution level coordinate ocean climate models. Here, the embedded Lagrangian model formulated in the companion paper of Bates et al. (2012) is used in two idealised test cases to examine the effect on the representation of dense gravity driven plumes, as well as the effect on the circulation of the bulk ocean in the Eulerian model. The results are compared with simulations with no parameterisation for overflows, as well as simulations that use traditional hydrostatic overflow schemes.

The use of Lagrangian “blobs” is shown to improve three key characteristics that are poorly represented in coarse resolution level coordinate models: (1) the depth of the plume, (2) the along slope velocity of the plume, and (3) the response of the bulk ocean to the bottom boundary layer. These improvements are associated with the more appropriate set of dynamics satisfied by the blobs, leading to a more physically sound representation. Experiments are also conducted to examine sensitivity to blob parameters. The blob parameters are examined over a large parameter space.

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction

In a companion paper (Bates et al., 2012, hereafter BGE-I), we describe the theory and implementation in the Modular Ocean Model (Griffies, 2012) of a dynamically interactive Lagrangian “blob” model. Here we focus on the ability of the Lagrangian blobs to improve the representation of gravity driven downslope flows in level coordinate ocean climate models. For this purpose, we test the bottom dynamically active implementation of blobs in two idealised test cases involving gravity driven downslope flows. Since the blobs explicitly represent elements of the physical processes absent in a purely coarse resolution Eulerian representation, it is hypothesised that use of the blobs will produce solutions that are more realistic. The test cases chosen also provide a very good platform to explore the parameter space of this new method. It is hoped that the results presented will help to guide future studies of overflows using the embedded Lagrangian model.

Blobs are parcels of seawater that are under the control of a Lagrangian sub-model and are treated quasi-independently of the Eulerian model. Blobs can be treated with a more appropriate set of dynamics than the hydrostatic primitive equations of the Eulerian ocean climate model. Dynamically active bottom blobs are formed when the difference in density between the shelf and the deepest grid cell in a neighbouring deep water column is greater than a certain threshold

$$\rho_{\text{shelf}} - \rho_{\text{deep}} > \Delta \rho,$$

where $\rho_{\text{shelf}}$ is the density of the onshelf water, $\rho_{\text{deep}}$ is the density of the bottom cell in the deep water column, and $\Delta \rho$ is a prescribed threshold.

Detailed derivations of the equations governing blob behaviour are given in BGE-I and Bates (2012). We restate these equations here for completeness. Blob trajectories evolve according to the momentum equations

\begin{align}
\vec{x} &= f \vec{y} + \partial_y H \left( \frac{g (\rho - \rho_s)}{\rho (H + z)} \right) - \frac{\tau_{(x)}^{(\text{bot})}}{\rho h_L} - \frac{\epsilon (\vec{x} - \vec{u}_k)}{h_L} \tag{2a} \\
\vec{y} &= -f \vec{x} + \partial_x H \left( \frac{g (\rho - \rho_s)}{\rho (H + z)} \right) - \frac{\tau_{(y)}^{(\text{bot})}}{\rho h_L} - \frac{\epsilon (\vec{y} - \vec{v}_k)}{h_L} \tag{2b} \\
\vec{z} &= g \left( \frac{\rho - \rho_s}{\rho} \right) \left( \frac{1}{\sqrt{(H + z)}} - 1 \right) - \frac{\tau_{(z)}^{(\text{bot})}}{\rho h_L} - \frac{\epsilon \dot{z}}{h_L} \tag{2c}
\end{align}
where \( H \) is the depth of the ocean (m), \( h_i \) is the height of the blob (m), \( f \) is the Coriolis parameter (s\(^{-1}\)), \( g \) is the acceleration due to gravity (m s\(^{-2}\)), \( \varepsilon \) is the entrainment velocity (m s\(^{-1}\)), \( \tau_{\text{bot}} = (\tau_{\text{bot}}^x, \tau_{\text{bot}}^y, \tau_{\text{bot}}^z) \) is the bottom stress (N m\(^{-2}\)), \( x = (x, y, z) \) is the position of the blob, and Newton's notation has been used for the material time derivative (e.g. \( \dot{x} = \frac{\partial x}{\partial t} \)). Using the parameterisation of Price and Baringer (1994), the bottom stress and entrainment velocity are given by

\[
\tau_{\text{bot}} = \rho C_d |\dot{x}| \times \varepsilon
\]

\[
\varepsilon = \begin{cases} 
|\dot{x} - u_i| & \text{if } Ri \leq 0.8 \\
0 & \text{if } Ri > 0.8 
\end{cases}
\]

\[
Ri = g (\rho - \rho_s) h_i / \rho |\dot{x} - u_i|^2,
\]

where \( Ri \) is the Richardson number and \( u_i = (u_k, v_k) \) is the Eulerian system's horizontal velocity. The exchange of tracer content between the Eulerian and Lagrangian systems is given by

\[
(C \frac{dm}{dt}) = (\rho_t C \dot{C} - \rho D) A_{\text{interface}},
\]

where \( C \) is the tracer concentration, \( D \) is the detrainment velocity (m s\(^{-1}\)), \( A_{\text{interface}} \) is the area of the interface between the blob and the Eulerian system and \( \rho_t \) is the density of the Eulerian system. The compatible expression for the rate of change of mass is given by

\[
\frac{dm}{dt} = (\rho_t \dot{C} - \rho D) A_{\text{interface}}.
\]

The detrainment velocity is given by

\[
D = - \frac{\Gamma}{|\rho - \rho_t|}
\]

where \( \Gamma \) is a parameter (kg m\(^{-2}\) s\(^{-1}\)) that is fixed and set \textit{a priori}.

The remainder of this paper consists of the following sections.

- **Section 2** examines previous process studies that have investigated the representation of overflows in level coordinate models at the resolution of global scale climate models. Particular attention is paid to studies that use the so-called DOME test case (e.g., Legg et al. (2006)) and the bowl test case (Winton et al., 1998), which are the two test cases used in this study.

- **Section 3** uses the DOME test case to examine the representation of a dense plume on a uniform slope. A fine resolution (~4.1 km) experiment is run as a benchmark. A coarse resolution setup is used to examine the effect of existing parameterisations and compared to the results obtained using the Lagrangian blob scheme. Parameter sensitivity to the bottom friction coefficient and the blob height is also investigated.

- **Section 4** uses the bowl test case to investigate the effect of overflowing waters on the large scale circulation. As with the DOME test case, the effect of existing parameterisations as well as the Eulerian system's friction scheme are examined and compared to the results obtained using the Lagrangian blob scheme. The parameter sensitivity to the detrainment parameter, and the blob initial conditions are also investigated.

- **Section 5** gives a summary and conclusions.

### 2. Earlier studies and parameterisations

There have been a number of idealised studies assessing the fidelity of overflows and gravity currents in ocean climate models. The purpose of this section is to summarise the studies that are directly pertinent to the present paper.

#### 2.1. Studies assessing level coordinate models in overflow regimes

The pioneering study of Winton et al. (1998) compared a z-coordinate model with an isopycnal model in an unforced relaxation experiment using a "bowl" topography. Deficiencies found in this and other studies have motivated much research into the accurate representation of overflows in ocean climate models and motivated various collaborations such as the Dynamics of Overflow, Mixing and Entrainment (DOME) and the Gravity Current Climate Process Team (Legg et al., 2009).

Legg et al. (2006) conducted a series of experiments at varying resolution in the DOME configuration with the MIT General Circulation Model (MITgcm) and the Hallberg Isopycnal Model (HIM). They examined the effects of changing resolution, slope, stratification, rotation and buoyancy anomalies. Their focus was predominantly on entrainment and diapycnal mixing in different vertical coordinate systems at varying resolutions. They found that in level models, coarse resolution models tend to have a lot of spurious mixing, however, in the range between coarse resolution and very fine resolution, the models may not have enough mixing. Other studies, such as that of Ezer and Mellor (2004) have shown that the representation of a dense plume is affected by the vertical coordinate and lateral diffusion, while Tseng and Dietrich (2006) showed that the results of a z-coordinate model are robust once the Rossby radius of deformation and the thickness of the plume are resolved.

#### 2.2. Overflow parameterisations in level models

Parameterisations of gravity driven downslope flows have been incorporated in level coordinate ocean climate models, aiming to address shortcomings. Examples include the sigma advection–diffusion scheme of Beckmann and Dösscher (1997), Dösscher and Beckmann (2000); the overflow scheme from Campin and Goosse (1999); and a slab bottom boundary layer (BBL) from Killworth and Edwards (1999) as well as Nakano and Sugino (2002). We summarise here elements of these schemes.

Beckmann and Dösscher (1997) embed a terrain-following sigma layer within a level model, with the sigma layer transporting only tracer fields and not momenta. Use of the advection portion of the sigma scheme was shown by Dösscher and Beckmann (2000) to be less important for coarse resolution ocean climate models, and so we focus here only on the diffusive portion of their scheme. The scheme prescribes an enhanced diffusion between bottom grid cells if conditions for downslope flow are density-favourable

\[
A = \begin{cases} 
A_{\text{max}} & \text{if } \nabla_a \rho \cdot \nabla H < 0 \\
A_{\text{min}} & \text{if } \nabla_a \rho \cdot \nabla H > 0 \end{cases}
\]

where \( A \) is the diffusion coefficient in the sigma layer with \( A_{\text{max}} \gg A_{\text{min}} \), and \( \nabla_a \) is the gradient taken along slope. The diffusive tracer flux between two adjacent bottom cells is thus given by

\[
F_a = -A \nabla_a C.
\]
directly transported from the shelf to the deep ocean without a specified return flow. Both schemes are tested in this paper.

Killworth and Edwards (1999) formulate a slab BBL scheme which alters both the momentum and tracer equations, and Nakano and Sugino (2002) implement a similar, though somewhat simpler, scheme. Both schemes exhibit relatively large numerical errors in the along slope pressure gradient. The pressure gradient error is particularly large at low latitudes due to the smallness of the Coriolis parameter. Even still, Nakano and Sugino (2002) show significant improvement in water mass properties formed in the northern North Atlantic, the Ross Sea and the Weddell Sea. In addition, the improved representation of overflows reduces spurious density gradients in those regions, which in turn reduces spurious effects associated with Gent and McWilliams (1990) scheme acting on those density gradients. We do not test these schemes here.

Bottom boundary layer currents are often important in regions where topography restricts flow (for instance, in the Greenland–Iceland–Norway Basin), with much of the outflow taking place through canyons that the present generation of ocean climate models are unable to resolve. One method widely used in coarse resolution models to capture such outflows is to unrealistically deepen topography. While unrealistically deeper topography may partly overcome the problems associated with not being able to represent outflows on the coarse grid topography, there are a number of undesirable effects associated with such a technique (see, for example, Roberts and Wood, 1997). To maintain a realistic topography while also capturing much of the effect of outflows, Kösters et al. (2005) diagnose the hydraulic transport over overflows from the large-scale conditions and impose a value of the exchange across a sill. They find some improvement in model realism. Born et al. (2009) conduct similar experiments using the same parameterisation and find that there is a large impact on the Atlantic meridional overturning stability for freshwater pulses, as well as a realistic improvement in the sub-polar gyre strength.

Some more recent developments are being incorporated in the present generation of ocean climate models. For example, modifications to boundary conditions to represent subgrid-scale straits and channels (e.g. the Strait of Gibraltar) include cross-land mixing (Griffies et al., 2005), as well as thin and partial barriers (A.J. Adcroft, personal communication, 2010). Having straits that are artificially widened, or completely closed, spuriously affects source water properties. Methods for improving the topographic representation of features works well for narrow channels and straits, however, they are unable to be utilised to represent undersea canyons and other subsurface topographic features, such as sea mounts.

Another approach defines a subsurface source region and a number of predefined subsurface injection locations (e.g. Danabasoglu et al., 2010). Water is taken from the source region and instantaneously transported to the injection location, where the product water is placed. The water may be modified based on properties of a predefined input region and entrainment region, which are between the source region and the injection locations. This technique relies on modification of topography by making the sill depth more shallow. The amount of source water transported from the source region, how much the water is modified, and the choice of injection location are all based on a physical model and calculated. The approach has been implemented for the Mediterranean overflow by Wu et al. (2007), with a more sophisticated version also used for other regions by Danabasoglu et al. (2010) and Briegleb et al. (2010). Danabasoglu et al. (2010) show that the parameterisation in the Nordic Sea overflows has important climate relevant effects. The largest effect was the removal of a spurious poleward deep western boundary current below about 2600 m, which almost universally improves the properties of the North Atlantic (Danabasoglu et al., 2010). A drawback of such a technique is the a priori assumptions required to setup the parameterisation, which limits the usefulness for experiments that are not of the present day climate (e.g. paleo-climate or future climate change simulations).

Although not directly related to overflow parameterisations, Ilicak et al. (2012) emphasised the importance of maintaining a near unit value for the grid Reynolds number. They suggested that a Laplacian Smagorinsky friction scheme (Smagorinsky, 1963; Griffies and Hallberg, 2000) is useful for this purpose, in which the flow dependent viscosity is given by

$$A_{SM} = (C_s \Delta s)^2 |E|$$

where $C_s$ is the non-dimensional Smagorinsky coefficient, $\Delta s$ is the horizontal grid spacing and $E$ is the total deformation rate with units of inverse time. In addition to the Smagorinsky viscosity, a background viscosity may also be defined

$$A_{\text{min}} = U_s \Delta s$$

where $U_s$ is a constant velocity scale (m s$^{-1}$). Given the importance of lateral friction in our overflow simulations, we illustrate the sensitivity of results in the idealised test cases to use of the Smagorinsky friction scheme.

3. The DOME test case

To investigate the representation of a plume under the influence of rotation on a uniform slope we conduct experiments using the dynamics of overflow, mixing and entrainment (DOME) test case. The DOME test case was developed for the specific purpose of testing overflows. A number of studies have used the configuration (e.g., Ezer and Mellor, 2004; Legg et al., 2006; Tseng and Dietrich, 2006; Legg et al., 2008). The DOME test case was used extensively by the Gravity Current Climate Process Team and for an updated review of the overflow problem, and the test cases such as DOME used to develop parameterisations, refer to the review paper by Legg et al. (2009).

The DOME configuration has cold, dense water in a shallow embayment which then flows down a uniform slope. The DOME test case is based loosely on a typical overflow (e.g. Denmark Strait Overflow, Mediterranean Outflow, etc.). Our implementation of the DOME test case has a couple of subtle differences with previous studies, which are outlined in Section 3.1.

3.1. Model configuration

The domain has a maximum depth of 3600 m and extends approximately 2000 km zonally and approximately 750 km meridionally. To the north of the domain there is a shelf with an embayment of depth $h_o = 600$ m approximately 2/3 the way along from the western wall. The height of the embayment is 300 m and the width of the embayment is approximately 81 km. From the embayment, there is a uniform slope of 0.01 m/m until the maximum depth. The topography of the domain is shown in Fig. 1. The domain has no slip lateral boundary conditions with solid walls at the edges.

In a departure from previous DOME studies, there is no mass flux at the northern end of the embayment, instead water is set to a constant temperature. Thus, the plume is purely density driven, and there is some recirculation required from the bulk ocean into the embayment to balance the cold water leaving. The reason why there is no mass flux prescribed is because we were unable to do so on Arakawa, 1966 B grid of MOM without inducing significant instabilities. It is only a portion of the northern edge of the embayment where the water temperature is set. The height, $h_o$, 62
of the region and the buoyancy anomaly relative to the initial conditions, \( b_i \) is defined as in previous DOME studies

\[
h_i(x_w) = h_0 \exp(-x_w/L_p)
\]

\[
b_i(x_w, z) = h_0 - \Delta h_0
\]

where \( x_w \) is the distance (in metres) from the western edge of the embayment. Also,

\[
L_p = \frac{\sqrt{h_0 \Delta h_0}}{f}
\]

is the deformation radius, \( h_0 = 300 \text{ m} \) is the height scale of the inflow, \( h_0 = h_0(z) \) is the initial buoyancy and \( \Delta h_0 = 0.019 \text{ m s}^{-2} \) is the buoyancy anomaly of the inflow.

Density is taken to be a linear function of temperature only

\[
\rho = \rho_0 - \alpha T
\]

where \( \alpha = 0.255 \text{ kg C}^{-1} \text{ m}^{-3} \) is a constant thermal expansion coefficient and \( T \) is temperature. A passive dye tracer is initialised to zero throughout the domain, but is set to one in the cells at the northern boundary (and hence no velocity specified) the Froude number can be assumed to be less than 1 (sub-critical). Thus, according to the classification of Cenedese et al. (2004) our experiments lie in the eddying regime (the eddy regime has an Ekman number less than 0.1), but, only weakly so and thus could be expected to display properties of either or both the eddying and laminar regimes.

Additional experiments (not shown) were conducted in the fine resolution case to test the sensitivity of the solution to viscosity. An experiment was conducted with a non-dimensional Smagorinsky parameter of \( C_s = 4.0 \pi \) and another with a vertical viscosity of \( 1 \times 10^{-4} \text{ m}^2 \text{ s}^{-1} \). An additional medium resolution \((\sim 70 \text{ km})\) experiment was run (not shown) to test the effect of resolution. The results of the experiment with the smaller vertical viscosity are consistent with what we would expect from having a smaller Ekman number, with a slightly shallower penetration and more eddy activity (Cenedese et al., 2004). Overall, however, the results of these sensitivity experiments do not have large deviations in bulk properties (e.g. plume depth, plume speed) with that of the fine resolution results presented here, particularly when compared to the coarse resolution experiments that do not use blobs.

### 3.2. Experiments

We present a series of experiments to examine the effect of a range of parameterisations in the DOME test case, with experiments listed in Table 1. The sigma and overflow experiments use the schemes of Beckmann and Dösch (1997) and Campin and Goosse (1999), respectively. In addition, effects of the modified Campin and Goosse (1999) scheme, introduced by BGE-I, are examined in the overflow experiment. We use the Eulerian framework for the modified Campin and Goosse (1999) scheme in the overflow experiment.

The objective of the experiments using dynamically active blobs is to ascertain the effect of blob parameters. The parameters under consideration are the bottom drag coefficient, \( C_d \), and the blob height, \( h_b \) – both of which directly affect the blob momentum Eqs. (2). The detrainment parameter, \( \Gamma \); the proportion of a grid
Table 1
Experimental details for all experiments in the DOME test case. $C_d$ is the dimensionless coefficient of drag for blobs, and $h_1$ is the prescribed height of blobs (in metres). $\sigma_{\text{overflow}}$ and $\sigma_{\text{nr}}$ are identical to $\sigma_{\text{noblob}}$ except they use the listed overflow scheme.

<table>
<thead>
<tr>
<th>Name</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>noblob</td>
<td>No overflow scheme</td>
</tr>
<tr>
<td>sigma</td>
<td>Beckmann and Döscher (1997) overflow scheme</td>
</tr>
<tr>
<td>overflow</td>
<td>Campin and Goosse (1999) overflow scheme</td>
</tr>
<tr>
<td>$\sigma_{\text{nr}}$</td>
<td>Modified Campin and Goosse (1999) overflow scheme</td>
</tr>
<tr>
<td>$\sigma_{\text{noblob}}$</td>
<td>Modified Campin and Goosse (1999) overflow scheme</td>
</tr>
<tr>
<td>ctrl</td>
<td>$C_d = 3 \times 10^{-3}$, $h_1 = 100$ m</td>
</tr>
<tr>
<td>free</td>
<td>$C_d = 3 \times 10^{-3}$, $h_1 = 100$ m</td>
</tr>
<tr>
<td>$c5e-2$</td>
<td>As in ctrl, only with $C_d = 3 \times 10^{-2}$</td>
</tr>
<tr>
<td>$c5e-4$</td>
<td>As in ctrl, only with $C_d = 3 \times 10^{-4}$</td>
</tr>
<tr>
<td>h500</td>
<td>As in ctrl, only with $h_1 = 50$ m</td>
</tr>
<tr>
<td>h200</td>
<td>As in ctrl, only with $h_1 = 200$ m</td>
</tr>
<tr>
<td>FR</td>
<td>No overflow scheme; fine resolution</td>
</tr>
</tbody>
</table>

These limitations we only conduct qualitative comparisons between the FR experiment and coarse resolution experiments.

3.3. Results and discussion

We are mostly concerned with the properties of overflowing waters in ocean climate models and their role in setting water-mass properties and their effect on the basin scale circulation, which are the known significant roles of overflows in the climate system (Legg et al., 2009). As such, we are interested in the large scale properties of the plume rather than fine scale details. Specifically, we focus on the depth of penetration of the plume, the along slope velocity of the plume and the density of the plume. Generally in coarse resolution models, the plumes tend to be too shallow, have a sluggish along slope velocity and be too diluted. These are not the only metrics by which to judge the fidelity of a simulation, however, we consider these to be important properties from the perspective of water-mass formation in ocean climate models (the effect on the basin scale circulation is discussed in Section 4).

A concept which assists in interpreting results is Nof (1983) speed, defined as,

$$U_N = \frac{\log g}{f}$$  \hspace{1cm} (13)

The Nof speed is the along slope speed of an anomalously dense parcel of water in the absence of friction. Here $\theta = 0.01$ m/m is the constant gradient of the slope, $f = 10^{-4}$ s$^{-1}$ is the constant Coriolis parameter and $g'$ is the reduced gravity. Thus, in our experiments, the Nof speed depends only on $g'$. The two major factors that affect the plume reduced gravity are plume density and plume depth. Density of the plume is set by the water density in the embayment, and how much mixing occurs as the plume travels along its pathway. The more mixing that occurs, the lower the plume density. Depth affects the reduced gravity because the plume is sinking into a stratified environment, with deeper plumes having smaller reduced gravity for a given plume density.

Using $\theta = 0.01$ m/m, $f = 10^{-4}$ s$^{-1}$ and the value of $g'$ for water at the north of the embayment, the resulting Nof speed is $U_N = 1.89$ m s$^{-1}$. So, in the absence of any mixing or sinking it would take roughly 5 days for the plume to travel the approximately 800 km from the western edge of the embayment to 6° E. Sinking, mixing and friction all slow down the progress of the plume, thus making 5 days a lower bound on the time for the plume to reach 6° E.

3.3.1. A qualitative assessment

To get an overall feeling for the nature of the solutions, we present here the passive tracer concentration of the bottom grid cells for a variety of experiments. The FR experiment is shown in Fig. 2 at day 40, which is approximately the time when the western edge of the plume reaches 6° E. The black line indicates the centre of mass of the plume. The plume is said to exist where there is at least one grid cell at a given longitude with a passive tracer concentration greater than 0.01; the same convention used by Legg et al. (2006). It can be seen that the plume takes approximately 40 days to reach 6°E and that, as would be expected, the plume travels along the slope under the influence of rotation, but, has a downslope component to its path.

The passive tracer concentration of the bottom grid cells is plotted for the noblob experiment in Fig. 3(a) and the ctrl experiment in Fig. 3(b). In Fig. 3(b) the latitude and longitude position of each blob in existence is also plotted as a blue dot. The solution for each experiment is plotted at the time when the plume is crossing 6°E, thus, the time that the snapshot is taken is different for the two experiments and is different to that of the FR experiment shown in Fig. 2.
Comparing the FR experiment, Fig. 2, with the noblob experiment in Fig. 3(a) shows how the noblob experiment displays properties that are typical for coarse resolution level coordinate models without an overflow parameterisation. The tracer distribution and centre of mass of the plume in the noblob experiment has a much greater along slope component than the FR experiment. The propagation of the plume along the slope is also much slower, as is indicated by the much later arrival time of the plume front at 6°E in the noblob experiment than the FR experiment. These solutions arising from differences in resolution are well known and commensurate with the results of previous numerical studies (e.g., Legg et al., 2006), while comparison with laboratory experiments (e.g., Cenedese et al., 2004) confirms that the shallow depth of the plume’s penetration in noblob is unrealistic.

While there remain differences in the detail between the ctrl (Fig. 3(b)) and FR experiments, it is clear that they are qualitatively much more similar than the noblob and FR experiments. As can be seen in Fig. 3(b), the ctrl experiment’s plume front reaches 6°E by day 28, while it takes about 180 days for this to occur in the noblob experiment. The plume footprint in the ctrl experiment does not hug the northern boundary as much as it does in the noblob experiment. It can thus be seen qualitatively that the blobs deepen the plume and increase its speed, thereby overcoming two of the major problems that traditionally afflict coarse resolution level coordinate models.

3.3.2. Traditional overflow schemes

The result of the replumbing of the Campin and Goosse (1999) scheme is that the overflow experiment’s plume penetrates deeper than that of noblob, as is shown in the evolution of mean plume depth in Fig. 4(a). The plume density of the overflow experiment is also slightly more dense than the noblob experiment, as can be seen in Fig. 4(b). Thus, the scheme of Campin and Goosse (1999) reduces the dilution problem and increases plume penetration. Deeper penetration of the overflow experiment’s plume in a stratified environment slows down the plume front relative to the noblob experiment. This slowdown can be seen by looking at the plume front position in Fig. 4(d). The reason for the slow down is that the Nof speed – see Eq. (13) – is smaller for the overflow experiment due to a smaller reduced gravity, Fig. 4(c). Thus, the effect on the reduced gravity of having a more dense plume is counteracted by the plume penetrating deeper.

The ovf_nr experiment, which uses the modified version of the Campin and Goosse (1999) scheme, is essentially a more extreme version of the overflow experiment. The mean plume depth is even deeper than in overflow and the density of the plume is even greater. However, the increased penetration of the plume diminishes the magnitude of the reduced gravity, Fig. 4(c), which significantly slows the along slope progress of the plume front, Fig. 4(d).

Fig. 4(a) shows that Beckmann and Döscher (1997) scheme in the sigma experiment increases the depth of penetration of the plume relative to noblob. Furthermore, Fig. 4(d) shows that the along slope progress of the western edge of the plume is much faster than in noblob. However, the faster along slope velocity cannot be due to an increased Nof speed, since sigma has a smaller magnitude of reduced gravity than in noblob. The enhanced along slope velocity in the sigma experiment is instead due to the scheme diffusing properties along the slope, as well as down the slope. The effect of this diffusion on bottom dye tracer concentration can be seen in Fig. 5 with the plume spreading out along the slope without regard to rotation or other dynamical considerations.
3.3.3. The Lagrangian blob scheme

The embedded Lagrangian blobs are now investigated in the DOME test case. Results of the ctrl experiment in Fig. 4(a) show that the blobs deepen the mean plume depth relative to the noblob experiment. The blobs also significantly increase the along slope velocity of the plume front, as can be seen by the rapid progression of the ctrl front in Fig. 4(d). The rapid progress of the ctrl plume is also clearly visible when comparing the bottom cell dye concentration of noblob at day 28, Fig. 6, with that of ctrl also at day 28, Fig. 3(b).

The reason for the faster along slope velocity when employing the embedded Lagrangian model is because the transport is being effected by the blobs themselves. The mean reduced gravity of the blobs in the ctrl experiment is shown in Fig. 7(c). As can be seen when comparing with the magnitude of the plume as a whole, Fig. 4(c), the reduced gravity of blobs in the ctrl experiment is significantly greater in magnitude than the reduced gravity of the whole plume. A larger value for the magnitude of the reduced gravity must either be due to the blobs not penetrating as deeply, or, the blobs being more dense.

The mean blob depth in the ctrl experiment is appreciably deeper than the mean plume depth; compare the red curves in Figs. 4(a) and 7(a). The blob density in ctrl is much greater than the density of the entire plume in the same experiment; compare the red curves in Figs. 4(b) and 7(b). This increased density explains why the reduced gravity of blobs in the ctrl experiment

---

Fig. 4. Plume properties that show the diagnostics that are used to assess the fidelity of the simulated plume. (a) The mean plume depth is an indication of the model’s ability to move material down slope. (b) The mean plume density is an indication of the amount of mixing occurring. (c) The mean plume reduced gravity is an indication of the Nof speed. (d) The position of the plume front (defined as the western edge of the plume) gives an indication of the along slope speed of the plume. The reduced gravity is calculated as $g' = \frac{g}{\rho_B}\frac{\rho_B}{\rho_P}$, where $\rho_B$ is the density of the plume and $\rho_P$ is the density of the bulk ocean. The experiments free, h050 and h200 largely track the ctrl experiment and have been omitted to reduce clutter.

Fig. 5. The dye concentration of the bottom cells and the centre of mass of the plume for the sigma experiment at day 50, when the plume front is at 6 E. The overflow scheme of Beckmann and Döscher (1997) increases the depth of penetration of the plume as well as the along slope velocity. This behaviour is achieved through enhanced diffusion, which is isotropic in this scheme meaning that the plume spreads out along the slope without regard to dynamical considerations such as rotation.

3.3.3. The Lagrangian blob scheme

The embedded Lagrangian blobs are now investigated in the DOME test case. Results of the ctrl experiment in Fig. 4(a) show that the blobs deepen the mean plume depth relative to the noblob experiment. The blobs also significantly increase the along slope velocity of the plume front, as can be seen by the rapid progression of the ctrl front in Fig. 4(d). The rapid progress of the ctrl plume is also clearly visible when comparing the bottom cell dye concentration of noblob at day 28, Fig. 6, with that of ctrl also at day 28, Fig. 3(b).

The reason for the faster along slope velocity when employing the embedded Lagrangian model is because the transport is being effected by the blobs themselves. The mean reduced gravity of the blobs in the ctrl experiment is shown in Fig. 7(c). As can be seen when comparing with the magnitude of the plume as a whole, Fig. 4(c), the reduced gravity of blobs in the ctrl experiment is significantly greater in magnitude than the reduced gravity of the whole plume. A larger value for the magnitude of the reduced gravity must either be due to the blobs not penetrating as deeply, or, the blobs being more dense.

The mean blob depth in the ctrl experiment is appreciably deeper than the mean plume depth; compare the red curves in Figs. 4(a) and 7(a). The blob density in ctrl is much greater than the density of the entire plume in the same experiment; compare the red curves in Figs. 4(b) and 7(b). This increased density explains why the reduced gravity of blobs in the ctrl experiment
is large compared to the reduced gravity of the plumes (compare Figs. 4(c) and 7(c)) despite the fact that the depth of the blobs is counteracting the density.

The reason why blobs have a much higher density than the rest of the plume is because their density is set at the edge of the embayment. The blobs are then able to maintain their density without significant dilution, because the transfer of seawater between the blobs and the Eulerian model is controlled by the entrainment/detrainment parameterisation.

3.3.4. Blob bottom friction

The c3e-2 and c5e-4 experiments are designed to examine the effect of the dimensionless bottom coefficient of drag, $C_d$, on the properties of the Lagrangian blobs by increasing and decreasing $C_d$ by an order of magnitude relative to the ctrl experiment. The position of the western edge of the plume, Fig. 4(d), indicates that the value of the coefficient of drag has a large effect on the plume speed. This is due to two factors. The first is that the coefficient of drag directly relates to the friction term in the blob momentum Eq. (2). A larger coefficient increases the retarding effect of bottom drag while a smaller coefficient decreases it. This direct effect on blob speed can be clearly seen in Fig. 7(d), which shows the evolution of the mean blob speed. The c3e-2 experiment, with $C_d = 3 \times 10^{-2}$, has much slower blobs than the ctrl experiment ($C_d = 3 \times 10^{-1}$), which in turn has much slower blobs than the c5e-4 experiment ($C_d = 3 \times 10^{-4}$).

The other reason why the progress of the plume along the slope is affected by the bottom blob coefficient of drag is because the drag term breaks geostrophic balance as water in geostrophic balance follows isobaths (Griffiths, 1986), which affects the depth of

---

**Fig. 6.** The dye concentration of the bottom cells and the centre of mass of the plume for the noblob experiment at day 28. The sluggish progress and shallow penetration of the plume in the noblob experiment is highlighted when compared to the ctrl experiment at the same time in Fig. 3(b).

**Fig. 7.** Average blob properties that show how their behaviour affects the plume. (a) The mean blob depth indicates the depth of penetration of the blobs. (b) The mean blob density shows how the blobs maintain their density. (c) The mean reduced gravity gives an indication of the Nof speed. (d) The speed of the blobs shows the actual speed of the blobs. Units are given in the y-axis labels. The reduced gravity is calculated as $g' = g(\rho_c - \rho_l)/\rho_s$. The free experiment largely tracks the ctrl experiment and has thus been omitted to reduce clutter.
the blobs. A larger coefficient of drag means that the angle that blob trajectories make with the isobaths is greater, thus, they penetrate deeper. This deeper penetration can be seen when comparing the mean blob depths of the c3e-2, ctrl and c3e-4 experiments in Fig. 7(a). Having blobs that penetrate deeper has two main consequences for the along slope velocity of the plume. The first is that blobs will reach their neutral depth more rapidly and will separate, thus, returning their properties to the Eulerian system or becoming free blobs. The second consequence is that the blobs will have a reduced gravity that is smaller in magnitude due to the deeper blob penetration. As a result the experiment with the highest value for $C_d$, c3e-2, has a smaller magnitude of reduced gravity than the ctrl experiment. The c3e-4 experiment has a larger reduced gravity than the c3e-2 experiment due to the shallower mean blob depth (Fig. 7(a)), but, a smaller reduced gravity than the ctrl experiment due to a smaller mean blob density (Fig. 7(b)).

When examining the position of the western edge of the plume in Fig. 4(d) it can be seen that the curves for the c3e-2 and c3e-4 experiments exhibit discontinuous behaviour. The reason for the discontinuity is that there can be longitudes where the combined passive tracer concentration of the E and L systems is less than 0.01, even when blobs are present. Blobs are destroyed when they interact with the western wall and the tracer is relaxed to the initial value by the sponge boundary conditions. Thus, gaps can occur in the plume and if relatively few small blobs are being formed the discontinuities seen in Fig. 4(d) can develop.

#### 3.3.5. Bottom blob height

To understand the effect of the blob height on the properties of blobs, it is useful to recall the momentum Eqs. (2), and how the bottom drag term and the interfacial drag term are inversely proportional to the height of a blob, $h_b$. A larger blob height, as in the h200 experiment ($h_b = 200$ m), is less encumbered by drag than a smaller value, such as the h050 experiment ($h_b = 50$ m). This behaviour is reflected in Fig. 7(d), which shows the mean speed of blobs. The relationship between height and the drag terms are not the only factor at play, however, since the ctrl experiment ($h_b = 100$ m) and the h200 experiment have approximately the same values from day 100 onwards, despite the h200 experiment having blobs that are twice the height as those in the ctrl experiment. We also note that the entrainment velocity, $\xi$, defined in Eq. (3b) is related to the Richardson number, $\xi$, defined in Eq. (3c). The mean blob Richardson number for the h200, ctrl and h050 experiments as a function of time is shown in Fig. 8. The value of the height of the blob, $h_b$, affects the mean value of the Richardson number, with a larger $h_b$ yielding a higher average value for $\xi$. This difference in Richardson number, however, does not translate into an appreciable difference in the entrainment velocity, $\xi$, with the ctrl, h200 and h050 experiments having no appreciable difference in entrainment velocity (not shown).

With the entrainment velocity being indiscernible between the experiments, the only explanation as to why there is such a difference between the experiments in the average blob density, Fig. 7(b), must be due to the density of the E system water in the main blob formation region (i.e. the source waters). Since the parameters that affect the initial conditions of the blobs are constant across the experiments in the DOME test case, the difference in density of the source waters must be due to variations in the circulation of the experiments and the mixing in the E system that occurs in and near the embayment.

#### 3.3.6. Blob separation from the bottom

In the ctrl experiment, blobs that satisfy the separation condition (i.e. the density of the blob is less than that of the surrounding E system) are destroyed. In the free experiment, a blob that satisfies the separation condition becomes a free blob and continues with the dynamics of a free blob (see BGE-I for details). In the DOME test case, we find this ability has a minor impact on the solution, with properties of the ctrl experiment and the free experiment being very similar throughout the simulations. An equivalent experiment was run in the bowl test case which also made very little difference to the results.

We hypothesise that the reason for the free blobs making very little difference is due to a combination of two factors. Firstly, when blobs separate they are generally only marginally positively buoyant. Thus, they have little impetus for vertical acceleration. Due to the blobs being close to their neutral buoyancy surface combined with the form of the detrainment velocity, Eq. (5), the blobs rapidly detrain. In essence the ctrl experiment instantaneously returns a blob’s properties to the E system while the free experiment returns its properties in approximately the same region over a finite amount of time. It is not clear to us that the employment of blob separation would have a similarly trivial effect in simulations with a more complex topography and density field than the DOME or bowl test cases.

#### 4. The bowl test case

The bowl test case was used in the study of Winton et al. (1998) to investigate the fidelity of the representation of gravity driven downslope flows. The hypsometric effect (Rhines and MacCready, 1989) plays a large role in the dynamics of the bowl test case. In the bowl test case, dense water descends the non-uniform slope in a cyclonic sense due to rotation. Friction breaks the geostrophic balance to provide a downslope component to the bottom boundary layer’s motion. The nature of the basin means that there is a convergence of mass in the bottom boundary layer in the centre of the basin. The convergence in the bottom boundary layer causes a divergence of the bulk ocean, which establishes an anti-cyclonic cell in response. For the divergence in the bulk ocean to balance the convergence in the boundary layer, the anti-cyclonic circulation of the bulk ocean needs to be much stronger than the cyclonic circulation of the boundary layer, since it is mostly viscous friction (which is much weaker than bottom friction) that breaks the
geostrophic balance in the bulk ocean. Thus, in the bowl test case the net circulation should be overwhelmingly anti-cyclonic and centred on the deepest part of the basin.

Winton et al. (1998) run a layered model and a level model, with the layered model capturing a strong, coherent anti-cyclonic flow centred on the deepest part of the basin, as predicted by hypsometric theory. Their level model on the other hand has a much weaker, less coherent anti-cyclonic circulation.

4.1. Model configuration

Our bowl test configuration is slightly different to that of Winton et al. (1998). As for the DOME test considered in Section 3, we choose an $f$-plane, centred on $43\,^\circ\text{N}$ with $f_0 = 10^{-4}$ s$^{-1}$, and $40(x) \times 41(y) \times 50(z)$ grid points corresponding to a grid spacing of approximately 40 km (roughly 1/2°). The grid cell thicknesses increase linearly from 20 m at the surface to 120 m at the bottom. The model domain spans from 0° to 20° (approximately 1700 km) zonally and from 35° to 50° meridionally (approximately 1300 km). The domain has a 500 m deep flat shelf to the north from 45°N. To the south of 45°N the topography is given by

$$H = 500 + 3000 \left\{ \exp\left(-\frac{(y-35)^2}{4}\right) \exp\left(-\frac{(y-45)^2}{4}\right) \exp\left(-\frac{x^2}{16}\right) \right\},$$

(14)

![Fig. 9. A wire frame of the bowl test case, based on the experimental design of Winton et al. (1998). The model uses no slip side boundary conditions (the sidewalls are not depicted here).](image)

as is shown in Fig. 9. Initially water on the shelf is $T = 15\,^\circ\text{C}$ while water to the south of 45°N is uniformly $T = 20\,^\circ\text{C}$. Density is taken to be a linear function of temperature only, as described by Eq. (12). The thermal expansion coefficient is constant at $\alpha = 0.255$ kg C$^{-1}$m$^{-3}$. The timestep for all experiments is 3600 s for tracer and baroclinic momentum, and 45 s for the barotropic system. There is no momentum or buoyancy forcing, and so the bowl test case is a pure relaxation simulation. The remainder of the configuration is identical to the DOME case considered in Section 3.

4.2. Experiments

All experiments are summarised in Table 2. The nomenclature of the DOME test case is maintained with the baseline experiment being the noblob experiment and the ctrl experiment being the control experiment for the blobs. The sigma experiment uses Beckmann and Döscher (1997) parameterisation and the ovf_nr experiment uses the modified version of Campin and Goosse (1999) experiment. Results from an experiment using Campin and Goosse (1999) scheme are not shown as the results are very similar to the noblob experiment.

All experiments have no lateral diffusion, with stability being ensured through numerical diffusion via the tracer advection scheme. The tracer advection scheme used in both the vertical and horizontal is the multi-dimensional piecewise parabolic method of Colella and Woodward (1984) with the flux limiter described by Lin (2004). No vertical diffusion is prescribed except for an enhanced diffusivity of 10 m$^2$ s$^{-1}$ in gravitationally unstable regions. The horizontal viscosity for each experiment is described in Table 2. A constant vertical viscosity of 0.5 m$^2$ s$^{-1}$ is used.

It has been suggested that the choice of friction scheme can have a large impact on the representation of overflowing waters (Ilicak et al., 2012). To examine this, the ksmag2 experiment uses the Laplacian Smagorinsky friction scheme (Smagorinsky, 1963; Griffies and Hallberg, 2000), with a non-dimensional Smagorinsky parameter of $C_s = 2.0\,\text{m}^2\,\text{s}^{-1}$ and a background viscosity using a velocity scale of $U_0 = 0.1\,\text{m}\,\text{s}^{-1}$ (see Section 2.2). The nomicom experiment is also conducted, in which the velocity scale of the background viscosity is set to zero, $U_0 = 0$ m s$^{-1}$ (i.e. there is no background viscosity).

Table 2 also includes the blob experiments that were conducted to examine the parameter space in the bowl test case. Particular attention has been paid to the three parameters that affect the initial blob size, the blob age and the number of blobs produced. Those parameters are the detrainment parameter, $\Gamma$, the fraction of a cell participating in an overflow event, $\delta$ and the density difference threshold at which a blob is formed, $\Delta \rho$. There is an additional experiment that is designed to examine the effect of having very small

**Table 2**

<table>
<thead>
<tr>
<th>Name</th>
<th>Viscosity</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>noblob</td>
<td>Constant, 500 m$^2$s$^{-1}$</td>
<td>No overflow parameterisation</td>
</tr>
<tr>
<td>ksmag2</td>
<td>Smagorinsky, $C_s = 2.0,\text{m}^2,\text{s}^{-1}$</td>
<td>No overflow parameterisation; $U_0 = 0.1$</td>
</tr>
<tr>
<td>nomicom</td>
<td>Constant, 500 m$^2$s$^{-1}$</td>
<td>Beckmann and Döscher (1997) overflow scheme</td>
</tr>
<tr>
<td>sigma</td>
<td>Constant, 500 m$^2$s$^{-1}$</td>
<td>The modified Campin and Goosse (1999) overflow scheme</td>
</tr>
<tr>
<td>ctrl</td>
<td>Constant, 500 m$^2$s$^{-1}$</td>
<td>$\Gamma = 1 \times 10^{-5}$, $\delta = 1.0$, $\Delta \rho = 0.005$</td>
</tr>
<tr>
<td>r100</td>
<td>Constant, 500 m$^2$s$^{-1}$</td>
<td>$\Gamma = 1 \times 10^{-5}$, $\delta = 1.0$, $\Delta \rho = 0.005$</td>
</tr>
<tr>
<td>d1b</td>
<td>Constant, 500 m$^2$s$^{-1}$</td>
<td>As in ctrl, only with $\delta = 1/3$</td>
</tr>
<tr>
<td>d1e-4</td>
<td>Constant, 500 m$^2$s$^{-1}$</td>
<td>As in ctrl, only with $\Gamma = 1 \times 10^{-4}$</td>
</tr>
<tr>
<td>d1e-8</td>
<td>Constant, 500 m$^2$s$^{-1}$</td>
<td>As in ctrl, only with $\Gamma = 1 \times 10^{-4}$</td>
</tr>
<tr>
<td>bblob</td>
<td>Constant, 500 m$^2$s$^{-1}$</td>
<td>Very large blobs</td>
</tr>
</tbody>
</table>
massive blobs formed by enforcing the creation of blobs that are 20% the mass of the grid cell of origin. This experiment, called $bblob$ (for big blob), typically has blobs that are an order of magnitude more massive than blobs formed in the $ctrl$ experiment.

The $r100$ experiment changes the density threshold at which a blob is formed from $\Delta \rho = 0.005 \text{ kg m}^{-3}$ in the $ctrl$ experiment to $\Delta \rho = 0.1 \text{ kg m}^{-3}$. The $dl13$ experiment reduces the value of $\delta$ from 1.0 as it is in $ctrl$ to 1/3, as is the value used in the original formulation of Campin and Goosse (1999). The detrainment parameter, $\Gamma$, is varied over a wide parameter space from $\Gamma = 1.0 \times 10^{-8} \text{ kg m}^{-2} \text{ s}^{-1}$ to $1.0 \times 10^{-6} \text{ kg m}^{-2} \text{ s}^{-1}$ in the $Gle-4, ctrl$, and $Gle-8$ experiments.

4.3. Results and discussion

In the following discussion, we make use of the barotropic quasi-streamfunction, $\psi$, to map horizontal volume transport. Since the free surface of MOM admits a divergent vertically integrated volume transport, it is not possible to use a streamfunction to diagnose horizontal maps of volume transport. Details of the derivation of the quasi-streamfunction are given in Appendix A.

4.3.1. A qualitative assessment

We begin by examining Fig. 10, which shows the bottom cell temperature and the barotropic quasi-streamfunction in the $noblob$ experiment at day 360. We can see from the bottom temperature that the dense water hugs the western boundary before being forced to turn eastward at the southern boundary. The barotropic quasi-streamfunction shows a largely dominant anti-cyclonic flow, with a number of weak cyclonic cells to the west and south of the domain. The weak cyclonic cells are coincident with the dense water making its way along the boundary. These weak cyclonic cells are eddies that are spinning off from the cold water mass travelling along the western boundary. The anti-cyclonic circulation here is much stronger and more coherent than the $0.4' \times 0.2' \sim 11 \text{ km}$ level model results of Winton et al. (1998), their Fig. 4a), but, is still much weaker than the $\sim 450 \text{ Sv} \left( 1 \text{ Sv} = 10^{6} \text{ kg s}^{-1} \right)$ circulation reported in their layered model (their Fig. 4b), also with a resolution of $0.4' \times 0.2' \sim 11 \text{ km}$. We believe that our level model results have a stronger anti-cyclonic circulation than that of Winton et al. (1998), despite being a much coarser resolution, because of differences in horizontal friction. As is shown in Section 4.3.2, the results of a purely Eulerian model are very sensitive to horizontal friction. Winton et al. (1998) use a biharmonic viscosity scheme with a constant coefficient of $-1 \times 10^{-6} \text{ m}^{2} \text{ s}^{-1}$, which we believe results in excessive spurious mixing due to the Ilicak et al. (2011) Reynolds number constraint being violated.

The bottom temperature and quasi-streamfunction of the Eulerian system is shown for the $ctrl$ experiment at day 360 in Fig. 11. The temperature distribution around the boundary is much more even after 360 days in $ctrl$ than it is in $noblob$. A snapshot of blob positions in $ctrl$ is shown in the bottom temperature plot as black dots. Initially, the blobs move around the bowl in a cyclonic direction, as can be seen from the blob positions at day 90 in Fig. 12. As the experiment continues the Eulerian system’s circulation strengthens and the interfacial drag term becomes more important in the blob momentum Eqs. (2), resulting in the alteration of blob trajectories toward the middle of the bowl, Fig. 11(a). It is important to reiterate that the barotropic quasi-streamfunction for $ctrl$ is for the Eulerian system only – it does not include a contribution from the Lagrangian system. Thus, the total flow for the combined system will be different to that indicated in Fig. 11(b). The quasi-streamfunction in $ctrl$ shows that the anti-cyclonic cell is significantly stronger and more coherent than in $noblob$.

The circulation in $ctrl$ is almost 3 times the strength of that found by Winton et al. (1998) in their layered model (their Fig. 4b). A contributing factor to such strong circulation is that there is a positive feedback established between the blob convergence and the anti-cyclonic cell. As noted above, the cell spins up in the opposite sense to the direction of travel of the blobs, which then, via interfacial drag, causes the blobs to have a much larger downslope component to their motion. This more direct blob pathway to the centre increases mass convergence, requiring a commensurate increase in Eulerian circulation to provide the necessary divergence. The stronger circulation then influences the blob path, and so the feedback continues. To reduce the blob convergence, $\Delta \rho$ can be increased, or, $\delta$ decreased (see Section 4.3.4). Indeed, the maximum cell strength for both $r100$ and $dl13$ are slightly more than 600 Sv at day 360 (not shown).

4.3.2. Sensitivity to lateral friction in the Eulerian system

The bottom cell temperature and barotropic quasi-streamfunction of the $ksmag2$ experiment is shown in Fig. 13. In contrast to the $noblob$ or $ctrl$ experiments, $ksmag2$ moves tracer down the slope so that the temperature of bottom cells in the deepest part of the domain are quite cool. The tracer initially travels along the western boundary, however, near the southwest corner the tracer starts to move directly down the slope. It is hypothesised that this response of the model is caused by the friction scheme being

![Fig. 10.](image-url) (a) Bottom cell temperature ($^\circ$C). (b) The barotropic quasi-streamfunction, $\psi$ (Sv).
affected by the changing slope of the topography, although the testing of this hypothesis is beyond the scope of this study.

Similar to the noblob experiment, the ksmag2 experiment spins off cyclonic eddies as dense water from the shelf moves southward along the western wall. At day 360 the quasi-streamfunction of the ksmag2 experiment has a relatively strong cyclonic cell centred on the plume of cold water near the bottom of the bowl, although, it is still significantly weaker than the strength of the cell in the layered model results of Winton et al. (1998). The cyclonic cell is the result of eddies that have spun off from cold, shallow water along the western boundary. So, while the ksmag2 experiment is effective at getting material down the slope, there is a relatively strong cyclonic circulation associated with the mass of water going down the slope. The strong cell at day 360 in ksmag2 is absent in nomicom (see Fig. 14), however, there are a number of relatively strong cyclonic eddies spun off earlier in nomicom (not shown). The bottom temperature distribution of the ksmag2 and nomicom experiments are qualitatively similar, thus, the nomicom experiment's bottom temperature is not shown. The Smagorinsky friction scheme assists in moving material down the slope, however, the resulting anti-cyclonic circulation is not as strong or coherent as the isopycnal results of Winton et al. (1998).

**Fig. 11.** (a) Bottom cell temperature and (b) the barotropic quasi-streamfunction of the ctrl experiment at day 360, which uses the dynamically active bottom blob scheme to represent overflows. The blobs (black dots in (a)) converging toward the centre of the domain induce a strong and coherent anti-cyclonic flow in the Eulerian system (b), as predicted by hypsometric theory (Rhines and MacCready, 1989). Depth contours are shown with values of 500–3000 m in 500 m increments. Note that the colour scale of the quasi-streamfunction differs with other figures.

**Fig. 12.** The bottom cell temperature (°C) and blob position (black dots) of the ctrl experiment at day 90. The blob trajectories are flowing in a cyclonic sense. Depth contours are shown with values of 500–3000 m in 500 m increments.

**Fig. 13.** (a) Bottom cell temperature and (b) the barotropic quasi-streamfunction of the ksmag2 experiment at day 360, which uses the Laplacian Smagorinsky friction scheme, but no overflow parameterisation. The change in friction scheme is effective at getting properties down the slope, however, this comes at a cost to the dynamics, with a relatively strong cyclonic cell appearing in the basin. Depth contours are shown with values of 500–3000 m in 500 m increments. Note that the colour scale of the quasi-streamfunction differs with other figures.
We hypothesise that this result is because the bottom boundary layer is still not properly represented in this model configuration.

4.3.3. Traditional overflow schemes

The Sigma experiment, which uses the overflow parameterisation of Beckmann and Döscher (1997) moves tracer along and down the slope via its diffusive nature, as can be seen in Fig. 15. From a dynamic perspective, the Sigma experiment has a much weaker (by a factor of about two) anti-cyclonic circulation than the Noblob experiment (not shown). Similar to the Noblob experiment, the Sigma experiment also has regions of cyclonic flow to the west and south of the domain. It can be concluded that, if anything, the application of the parameterisation of Beckmann and Döscher (1997) degrades the simulation because it weakens the circulation and produces an unphysical temperature distribution.

In contrast to the Sigma experiment, the Ovf_nr experiment has a relatively strong anti-cyclonic flow centred on the middle of the basin, without any cyclonic cells to the west or south of the domain (Fig. 16). The stronger circulation arises because the modified Campin and Goosse (1999) scheme explicitly moves mass down the slope, whereas the standard scheme only shuffles tracer. The convergence of mass that is moved down the slope by the parameterisation in the Ovf_nr experiment spins up a relatively strong anti-cyclonic cell. While the response of the bulk ocean in Ovf_nr matches hypsometric theory more closely than in Noblob, the physics of the bottom boundary layer is still not properly captured by the modified Campin and Goosse (1999) scheme. For instance, water flowing down the slope is predicted by hypsometric theory to do so in a cyclonic sense (Rhines and MacCready, 1989), while the modified Campin and Goosse (1999) scheme just moves seawater toward the centre of the basin. Thus, a physically based representation of the bottom boundary is lacking in the Ovf_nr experiment.

4.3.4. Blob initial conditions

The density difference threshold, $\Delta \rho$, controls what the density contrast between an on-shelf grid cell and the bottom of the neighbouring deep water column must be before a blob is created. The Ctrl experiment requires that density of the shelf cell is greater than the density of the off-shelf cell by $\Delta \rho = 0.005$ kg m$^{-3}$. The r100 experiment has a much larger value of $\Delta \rho = 0.1$ kg m$^{-3}$. The impact of increasing the density threshold is to reduce the number of new blobs formed, as can be seen in Fig. 17(a). The reason for the formation of fewer blobs in r100 is that the density difference required for blob formation is attained much less frequently. Another consequence of having a larger density threshold is that larger blobs tend to be formed, as can be seen in Fig. 17(b). The reason is that the density contrast is part of what makes up the calculation of the initial mass of a blob

$$m \propto \Delta \rho$$

(see Eqs. (42)–(44) of BGE-I).

The other factor that affects the initial mass of blobs is based on the $\delta$ parameter of Campin and Goosse (1999), which is the proportion of a grid cell that is participating in an overflow event, which is...
chosen *a priori*. The $d_{13}$ experiment has $\delta = 1/3$ while the $ctrl$ experiment has a value of $\delta = 1.0$. Fig. 17(b) shows that the blobs in $d_{13}$ are consistently less massive than those in $ctrl$.

The evolution of the number of blobs in each simulation is shown in Fig. 17(c). Given that the $d_{13}$ experiment has smaller blobs than $ctrl$ and the $r_{100}$ experiment has larger blobs, it might be tempting to conclude that there is a relationship between the mass of a blob and the number of blobs. However, the $d_{13}$ experiment creates fewer new blobs than the $ctrl$ experiment for most of the duration of the experiment, as is shown in Fig. 17(a). This indicates that the $ctrl$ experiment is destroying more blobs than the $d_{13}$ experiment. The reason is that $ctrl$ violates the grid cell mass constraint (see the discussion in Section 4.3.5) more readily due to the combination of the number of blobs, the size of the blobs and the shape of the topography. The topography is a contributing factor here because it causes a convergence of blobs toward the centre of the bowl. The $r_{100}$ experiment, on the other hand, has far fewer blobs than the $ctrl$ experiment (Fig. 17(c)) despite the blobs in $r_{100}$ being larger; the smaller number of blobs means the grid cell mass constraint is violated less frequently because there are fewer blobs formed (see Fig. 17(a)).

4.3.5. The creation of very massive blobs

The $bblob$ experiment enforces the creation of very massive blobs. It does so by forming blobs that are one-fifth the mass of the grid cell of origin. A limitation of the implementation in MOM is that bottom blobs are considered to reside entirely in the bottom most grid cell. Thus, when a relatively small number of very massive blobs reside in a single bottom cell, the grid cell mass constraint (see Section 3.1 of BGE-I) is readily violated and in such instance blobs must be destroyed. In the implementation, older blobs are destroyed in preference to younger blobs (see Section 6.5.3 of Bates (2012) for details). A consequence of the grid cell mass constraint and the order of destruction is that most blobs in the $bblob$ experiment are destroyed very soon after they are created. The reason is that blobs are formed at such a rate that few are able to get away from the immediate area of formation prior to the grid cell mass constraint being violated by the formation of new blobs. Given that the blobs have a very short lifetime, the parameterisation largely behaves like the modified Campin and Goosse (1999) scheme. The simulation thus has the same drawbacks as identified in the $ovf_{nr}$ experiment. Namely, while material is being transported toward the centre of the bowl, the physics of the bottom boundary layer is effectively unresolved.

4.3.6. Detrainment parameter

The primary effect of the detrainment parameter, $\Gamma$, is to control the rate at which blobs detrain. In the absence of other effects, large values for $\Gamma$ should lead to short blob lifespans. This behaviour can be seen when comparing the average age of blobs in the $ctrl$ experiment ($\Gamma = 1.0 \times 10^{-6}$ kg m$^{-2}$ s$^{-1}$) and the $G_{1e-4}$ experiment ($\Gamma = 1.0 \times 10^{-4}$ kg m$^{-2}$ s$^{-1}$) in Fig. 17(d). The $G_{1e-4}$ experiment has an appreciably younger average blob age than $ctrl$. 

---

Fig. 17. Average blob properties that show how the blob parameters affect the evolution of the Lagrangian system. (a) The number of new blobs per timestep shows how many blobs are formed each time step. (b) The blob mass indicates how large the blobs are; the number of blobs. (c) The number of blobs shows how many blobs there are in existence at any given time. (d) The age of blobs indicates how long blobs exist for.
We can also see that $G_{-8}$ has a younger average blob age than $c_{tr}$ despite having a detrainment parameter that is two orders of magnitude smaller ($F = 1.0 \times 10^{-8}$ kg m$^{-2}$ s$^{-1}$). The reason is that the blobs in $c_{tr}$ and $G_{-8}$ have many more blobs destroyed due to the violation of the grid cell mass constraint when compared to the blobs in $G_{-4}$. In the bowl test case the main mechanism for destruction of blobs in $G_{-4}$ is due to blobs fully detraining, while in $c_{tr}$ and even more so in $G_{-8}$, the main mechanism is due to the violation of the grid cell mass constraint.

A consequence of having blobs detraining rapidly is that $G_{-4}$ has far fewer blobs than $c_{tr}$ (Fig. 17(c)). On the other hand, because $c_{tr}$ and $G_{-8}$ have similar mechanisms for destruction of blobs, the number of blobs in those experiments is similar. A large detrainment parameter will generally lead to fewer blobs, however, there is some value (dependent on the topography of the experiment itself) below which the solution is relatively insensitive to $F$ because the dominant mechanism for destroying blobs becomes the grid cell mass constraint, rather than blobs fully detraining.

5. Discussion and conclusions

The dynamically active bottom blobs of the embedded Lagrangian model, as formulated by Bates et al. (2012), BCE-4, have been tested in a series of overflow experiments. The dynamically active bottom blobs are designed to better represent gravity driven downslope flows, which are an important process for the meridional overturning circulation and the formation of the ocean's deep and bottom waters. Two test cases were used to assess the model at a resolution that is comparable to the present generation of global scale ocean climate models. One test case focused on the representation of the plume itself, while the other focused on circulation of the bulk ocean in response to overflowing waters. The results using the embedded Lagrangian scheme were compared to results using existing overflow parameterisations.

Coarse resolution level coordinate models are well known for having plumes that are spuriously diluted, too shallow and very sluggish. In the DOME test case (Section 3) we examined the representation of a gravity driven plume on a uniform slope. We found that Campin and Goosse (1999) parameterisation and a modified version (described by BCE-4) increases the depth of penetration of the plume, but this result comes at the detriment of the along slope plume speed, with the parameterisations making the plume even more sluggish. The embedded sigma scheme of Beckmann and Döscher (1997) improves both the penetration depth of the plume and the along slope velocity. The mechanism that the parameterisation uses to achieve this behaviour is diffusion between bottom grid cells. This diffusion pays no attention to dynamic considerations such as rotation, which leads to unphysical behaviour such as plume propagation in both directions along the slope. The Lagrangian blob scheme in contrast allows for deeper penetration of the plume and an increased plume speed. The blobs also better maintain the properties of the source waters, with the dilution of the blob’s density through parameterised entrainment rather than spurious numerical mixing. The improvements documented here indicate that the embedded Lagrangian model offers a viable route to significantly improve how gravity driven downslope flows are represented in ocean climate models. We feel that the parameter values for the $c_{tr}$ experiment in the DOME test case represent sensible choices for that particular test case.

The second test case utilised the bowl configuration of Winton et al. (1998), which examined the response of the Eulerian model to overflow parameterisations as well as the Eulerian lateral friction scheme, which has been suggested to have a large impact on properties of overflows (Ilicak et al., 2012). The bowl test should physically be dominated by the hypsometric effect (Rhines and MacCready, 1989), where dense water flowing down the slope of a bowl shaped basin induces a strong anti-cyclonic response in the bulk ocean, centred over the bottom of the bowl. For a level model with a constant uniform viscosity, material does not move down the slope very effectively and Winton et al. (1998) found that the bulk ocean response is weaker and less coherent than the results of an equivalent layered model. The parameterisation of Beckmann and Döscher (1997) merely spreads tracer around the basin and weakens the circulation which, by these diagnostics, means that the scheme is performing worse than experiments including no parameterisation. The Campin and Goosse (1999) scheme has a negligible impact on the solution. In contrast, the modified Campin and Goosse (1999) scheme increases the coherence and strength of the bulk ocean circulation, although, the physics of the bottom boundary layer are still absent. The Laplacian Smagorinsky friction scheme, without any overflow parameterisation moves material down the slope more effectively, however, the response of the bulk ocean is still not what would be expected from hypsometric theory. The Lagrangian blobs in contrast transport material down the slope, explicitly representing the bottom boundary layer. The bulk ocean responds to the convergence of mass of the blobs toward the centre of the bowl with a strong and coherent anti-cyclonic cell. This behaviour is consistent with hypsometric theory.

The embedded Lagrangian model thus improves what has long been considered a weakness of coarse resolution level coordinate ocean climate models: namely, overflows. This improvement is achieved through a Lagrangian discretisation of gravity driven downslope flows, thereby facilitating the admission of more appropriate physics. Not only does the Lagrangian scheme directly improve the representation of a dense plume, but the ability to interact with the Eulerian model in a physically based way also indirectly improves the physics of the model.

The results presented here provide strong motivation for future studies to examine the effect of dynamically active bottom blobs in realistic, global scale ocean climate model simulations. The model code underpinning this two-part study has been made publicly available to facilitate these global-scale ocean modelling efforts and is available from http://www.gfdl.noaa.gov/fms. The results presented here on the parameter sensitivities provide a useful starting point for the selection of parameter values for these future studies.

Acknowledgements

Thanks to Sonya Legg for making herself available to answer questions about the DOME test case. Thanks go to Trever McDougall for hosting MLB and SMG at CSIRO Marine and Atmospheric Research in Hobart during the first half of 2011. We wish to thank Sonya Legg and Mehmet Ilicak for their helpful comments which improved this manuscript. We would also like to thank the two anonymous reviewers for their helpful and constructive comments which improved the manuscript. This work was supported by an award under the Merit Allocation Scheme on the NCI National Facility at the ANU. This study was supported by the Australian Research Council. MLB is grateful for support from the ARC Centre of Excellence for Mathematics and Statistics of Complex Systems and the ARC Network for Earth System Science.

Appendix A. The barotropic quasi-streamfunction

The rigid lid method (Bryan, 1969) enforces a non-divergent vertically integrated velocity

$$\nabla \cdot \mathbf{U} = 0.$$  (A.1)
where the vertically integrated velocity is given by
\[ U = \int_{z_a}^{z_b} \mathbf{u} \, dz, \quad (A.2) \]
which has units m² s⁻¹. Hence, the flow can be described by a streamfunction
\[ \mathbf{U} = \mathbf{z} \wedge \nabla \psi \quad (A.3) \]
where \( \mathbf{z} \) is the vertical unit vector and \( \hat{\psi} \) is the streamfunction. The difference in value of the streamfunction between two points \( a \) and \( b \) is the volume transport (m³ s⁻¹) between those points
\[ \mathbf{V}_{ab} = \int_{z_a}^{z_b} \mathbf{U} \, dz = \psi_b - \psi_a, \quad (A.4) \]
where \( d \ell \) is a line element along any path connecting the points \( a \) and \( b \), \( \mathbf{n} \) is a rightward pointing unit vector perpendicular to the direction of integration along \( d \ell \). Typically the volume transport is written with units of Sverdrups where 1 Sv = 10⁶ m³ s⁻¹. In the case of a divergent flow in a non-Boussinesq fluid, the vertically integrated continuity equation is given by
\[ \nabla \cdot \mathbf{U}^r = -\partial_r \left( \int_{z_0}^{z} \rho \, d \zeta \right) + Q_w \quad (A.5) \]
where \( Q_w \) is the mass flux (kg m⁻² s⁻¹) of water crossing the surface boundary, \( \eta \), and the vertically integrated density weighted velocity, \( \mathbf{U}^r = (U^r, V^r) \), is given by
\[ \mathbf{U}^r = \int_{z_0}^{z} \rho \, dz \quad (A.6) \]
which has units of kg m⁻¹ s⁻¹. The divergent nature of the vertically integrated continuity Eq. (A.5) means that in order to describe \( \mathbf{U}^r \), a velocity potential, \( \chi \), is required in addition to the streamfunction
\[ \mathbf{U}^r = \mathbf{z} \wedge \nabla \hat{\psi} + \nabla \chi \quad (A.7) \]
For a Boussinesq, rigid lid model with zero surface water fluxes the gradient of the velocity potential vanishes, giving Eq. (A.3). Unlike the non-divergent volume transport (A.4), we are unable to calculate the mass transport of a non-Boussinesq, horizontally divergent flow directly from the streamfunction
\[ M_{ab} = \int_{z_a}^{z_b} \mathbf{n} \cdot \mathbf{U} \, d \ell, \quad (A.8) \]
where \( M \) is the mass transport (kg s⁻¹). Typically the mass transport is written with units of Sverdrups where 1 Sv = 10⁶ kg s⁻¹. While accurate, the integral (A.8) does not readily provide a horizontal map of transport. This motivates us to map the function
\[ \psi = -\int_{y_0}^{y} U^r dy' \quad (A.9) \]
where the lower limit, \( y_0 \), is usually taken as the southern boundary of the domain (which is solid in realistic global simulations). Thus, by definition
\[ \partial_y \psi = U^r. \quad (A.10) \]
We note, however, that the zonal derivative does not generally yield the meridional mass transport
\[ \partial_x \psi \neq V^r, \quad (A.11) \]
due to the divergent nature of the vertically integrated flow. It is for this reason that we call \( \psi \) a quasi-streamfunction. The choice of (A.9) is not unique, and other possibilities exist, for example
\[ \psi^* = \psi(x_0) + \int_{x_0}^{x} V^r dx'. \quad (A.12) \]
The difference between \( \psi \) and \( \psi^* \) gives an indication as to the magnitude of the divergence of the field. By and large the divergence of most simulations is small, particularly when long time averages are taken. The calculation of the quasi-streamfunction, \( \psi \), is therefore a useful measure of the mass transport. For the above, we can regain the Boussinesq approximation (which is relevant to the \( x \) vertical coordinate used in this study) by setting \( \rho = \rho_0 \) to a uniform constant.

References


