Stochastically Forced Modes of Interannual Southern Ocean SST Variability in a Simple Coupled Model

Christopher M Aiken* and Matthew H England

Centre for Environmental Modelling and Prediction, School of Mathematics,
University of New South Wales, Sydney, New South Wales, Australia

Submitted to Journal of Climate (NOTES AND CORRESPONDENCE)

* Corresponding author address: Chris Aiken, School of Mathematics, UNSW, Sydney
2052, NSW, Australia.
E-mail: aiken@maths.unsw.edu.au
ABSTRACT

A simple linearised coupled model of the Southern Ocean and overlying atmosphere is analysed for existence of modes of variability reminiscent of the Antarctic Circumpolar Wave (ACW). Contrary to a previous analysis of the model, no modes were found that match the observed propagation characteristics of the ACW. The most unstable mode of the system represents the westward propagation of a free ocean mode, which drives sea surface temperature (SST) anomalies at the same frequency. A least stable ACW-like coupled mode is only found when the ocean modes are heavily damped. In this case the system is effectively uncoupled and the least stable mode simply corresponds to the advection of SST anomalies. White-noise forcing of the SST advection equation, representing stochastic variations in the anomalous meridional ocean surface velocity of order 5 cm s\(^{-1}\) with a mean meridional SST gradient of 0.4 °C (° lat.)\(^{-1}\), produces SST anomalies of order 0.5 °C, comparable to the amplitude of the ACW. The dominant response frequency is also consistent with the ACW for reasonable choices of the model parameters. When extended to allow zonal variability the model produces a low wavenumber interannual response of amplitude 1 °C. In the long term mean variance was broadly spread among low wavenumbers, although single dominant wavenumbers occur over individual periods of decades. These results are consistent with the notion that a simple stochastically-forced advection model of SST anomalies can explain interannual Southern Ocean SST variability to leading order.
1 Introduction

Observational evidence suggests the existence of modes of interannual variability in a number of the ocean's basins. The most studied and best understood of these is the ENSO phenomenon in the tropical Pacific, but modes of superannual variability have also been documented in the North Atlantic, North Pacific, and Southern Ocean. The most heralded mode of interannual variability in the Southern Ocean is the Antarctic Circumpolar Wave (ACW), first observed in sea surface temperature (SST), sea level pressure (SLP), surface winds and sea ice extent by White and Peterson (1996), and in sea surface height by Jacobs and Mitchell (1996). The ACW has been characterised from observations as an eastwards propagating set of phase-linked anomalies in the above variables, with dominant wavenumber 2 spatial pattern and a period of around 4 years. Owing to the limited amount of data available for the analysis of White and Peterson (1996), the robustness of the ACW characteristics has been a topic of some debate. However, a number of numerical models have exhibited ACW-like variability in SST and sea surface density (eg Christoph et al 1998, Weisse et al 1999, Bonekamp et al 1999, Cai et al 1999), although with a number of differences to the White and Peterson (1996) analysis.

While ENSO is a true coupled mode of the atmosphere/ocean system, the generating mechanism for the extratropical modes of variability such as the ACW remains less clear. The ACW observations of White and Peterson (1996) show a fixed phase relationship between SST and SLP, suggestive of a coupled mode. Qualitatively plausible feedback mechanisms have been proposed that are consistent with the observed phases, in which high (low) SLP anomalies drive positive (negative) SST anomalies, which in turn reinforce the original SLP through anomalous heating. The studies of Qiu and Jin (1997) (hereafter QJ97), Talley (1999), and Baines and Cai (2000) found that self-sustaining coupled
modes with the phase relationship of the ACW observations existed in simple coupled models of the Southern Ocean. In the case of QJ97 and Baines and Cai (2000) the fastest growing modes were found to share many of the characteristics of the ACW from White and Peterson (1996). Talley (1999) concluded that an ocean with a Sverdrup response to wind-stress coupled to an atmosphere with a Sverdrup response to heating-induced vertical advection best matches the observed phase relationship. In addition, White et al (1998) demonstrated that atmosphere-ocean coupling was necessary to sustain ACWs that match the observed propagation characteristics in a coupled model of the global lower troposphere and upper ocean.

There exists doubt, however, that atmospheric heating in the extratropical oceans is sufficiently strong to sustain true coupled modes, and that the observed variability is more likely dominated by atmosphere to ocean forcing. Realistic global coupled models have produced ACW-like responses in Southern Ocean SST (Christoph et al 1997, Weisse et al 1999, Bonekamp et al 1999, Cai et al 1999), but without the observed wavenumber and phase relationship with SLP. The interannual SST variability in a number of these models was shown to be well described to leading order by simple forced advection models, and hence the existence of a fully coupled mode was not necessary to explain the interannual variability. These studies also found that the SST variability in their models was predominantly determined by local atmospheric forcing rather than atmospheric teleconnection. QJ97 also suggest this to be the case for the ACW from analysis of the propagation of anomalous atmospheric pressure from the tropics to the midlatitude Southern Hemisphere.

A number of authors have shown that a coherent integrated ocean response is possible under essentially stochastic forcing, by-passing the notion of exponentially growing modes (e.g. Saravanan and McWilliams 1997, Hasselman 1976, Frankignoul and Reynolds 1983). A related approach has been to understand ocean/atmosphere variability as modal in
nature, but sustained by stochastic forcing rather than through linear instability of the
modes themselves (eg Jin 1997, Penland and Sardeshmukh 1995, Griffies and Tziperman
1995, Kleeman and Moore 1997). In this study we use the latter approach to investigate the
response of the simple coupled model of QJ97 to stochastic forcing, under the hypothesis
that interannual SST fluctuations in the Southern Ocean may be in part described through
consideration of stochastically-forced damped linear modes.

2 A linearised coupled model of Southern Ocean SST
anomalies

The model used here is based heavily on that of QJ97. This model was selected as being the
simplest to possess ACW-like modes. The more comprehensive linearised coupled model
of Baines and Cai (2000) will be analysed in a later study. The physics and parameters in
the present model are identical to that of QJ97. The only significant difference is in the
form of the ocean component of the model; whereas QJ97 solve for geopotential, here we
use the more standard formulation in terms of streamfunction. The ocean component of
the model consists of two quasigeostrophic layers linearised about a mean ocean state of
purely zonal flow \((U_1, U_2)\):

\[
(\partial_t + U_1 \partial_x) [\nabla^2 \psi_1 - \frac{f_0^2}{g H_1} (\psi_1 - \psi_2)] + \psi_{1x} (\beta + \frac{f_0^2}{g H_1} (U_1 - U_2)) = \frac{1}{H_1} \text{curl} (\tau) \tag{1}
\]

\[
(\partial_t + U_2 \partial_x) [\nabla^2 \psi_2 - \frac{f_0^2}{g H_2} (\psi_2 - \psi_1)] + \psi_{2x} (\beta + \frac{f_0^2}{g H_2} (U_2 - U_1)) = 0. \tag{2}
\]

Here \(\psi_n\) represents the anomalous streamfunction in layer \(n\), \(g'\) is the reduced gravity,
\(H_n\) is the resting thickness of layer \(n\), \(f_0\) is a central value of the Coriolis parameter,
\(\beta = \partial_y f|_{f = f_0}\), \(\text{curl}(\tau)\) is the wind stress curl, and \(\partial\) represents the partial derivative
with respect to the subscript variable. A number of studies have demonstrated the success

The evolution of anomalous SST ($T$) in the model is controlled by upper layer ocean advection and thermal damping:

$$T_t = -U_1 T_x - J(\psi_1, T^*) - \kappa_o T,$$

where $T^*$ is the mean SST field, $J(a, b) = a_y b_x - a_x b_y$ is the Jacobian and $\kappa_o$ is the ocean thermal damping rate. Assuming $T^*$ is also purely zonal, then

$$T_t = -U_1 T_x - \psi_1 x T^*_y - \kappa_o T. \quad (3)$$

Additional contributions to the SST tendency from Ekman pumping and atmospheric heat flux may be incorporated into the model. However, for simplicity and consistency with QJJ97, in the following we consider only the effect of meridional advection of mean SST in the model. It may be noted that inclusion of an Ekman pumping term of the form $\text{curl}(\tau) T^*_z / \rho_o f_o$, with $T^*_z = 0.01 \degree \text{Cm}^{-1}$ in the model made only a negligible difference to the results that follow.

The system is closed with a heat balance atmosphere. The equation for anomalous atmospheric temperature for a purely zonal mean temperature $T^*_a$ and wind field $U_a$ is:

$$U_a T^*_{ax} + v_a T^*_{ay} + w_a T^*_{az} = -\kappa_a T_a + b \kappa_o T,$$

with $v_a$ and $w_a$ the meridional and vertical wind anomalies, $\kappa_a$ the atmospheric thermal damping rate, and $b$ the conversion coefficient for thermal forcing. In this model the response of the atmosphere to the oceanic heat flux is parameterised as a change in the SLP anomaly $p$ through:

$$p = \lambda T^*_a.$$
That is, the atmosphere is assumed to be equivalent barotropic, a justifiable assumption at high latitudes (Karoly 1988). If the vertical advection term is assumed to be dominated by the horizontal advection term, and \( v_a \) is related to \( p \) through geostrophy \( v_a = p_x/(f_0\rho_a) \), then the equation for \( p \) is:

\[
(U_a + \lambda/(f_0\rho_a)T_{ay})p_x = -\kappa_a p + b\lambda\kappa_a T. \tag{4}
\]

The wind stress curl used to force the ocean model is deduced from the pressure field:

\[
curl(\tau) = \alpha \rho_a/(\rho_a f_0) \nabla^2 p, \tag{5}
\]

where \( \alpha \) is the drag coefficient.

In the model the ocean is forced by wind stress curl determined from the pressure field. The inclusion of two layers permits lower frequency baroclinic waves, and this was found to be an important element by QJ97. SST is forced purely by the upper-level ocean velocity field through meridional advection of the mean temperature gradient. The atmospheric pressure is diagnostic, being completely determined by the SST anomaly. The exact parameter values of QJ97 were used to simulate the Southern Ocean and its overlying atmosphere. These are: \( f_0 = -1.19 \times 10^{-4} \text{ s}^{-1} \), \( \beta = 1.32 \times 10^{-11} \text{ s}^{-1} \text{ m}^{-1} \), \( H_1 = 500 \text{ m} \), \( H_2 = 4500 \text{ m} \), \( g' = 0.015 \text{ m s}^{-2} \), \( \rho_0 = 1000 \text{ kg m}^{-3} \), \( \rho_u = 1.23 \text{ kg m}^{-3} \), \( U_1 = 0.12 \text{ m s}^{-1} \), \( U_2 = 0.08 \text{ m s}^{-1} \), \( U_a = 10.0 \text{ m s}^{-1} \), \( T_y = T_{ay} = 0.4^\circ \text{C (o-lat.)}^{-1} \), \( \kappa_o^{-1} = 12 \text{ months} \), \( \kappa_a^{-1} = 2 \text{ weeks} \), \( \epsilon = 0.9 \times 10^{-5} \text{ m s}^{-1} \), \( b = 134 \) and \( \lambda = 200 \text{ Pa (oC)}^{-1} \). The channel dimensions are \( L_x = 23000 \text{ km} \) and \( L_y = 1100 \text{ km} \), approximately \( 360^\circ \text{ lon. by 10^\circ \text{ lat. at latitude 55^\circ S}} \).

Solutions to the system of equations (1) - (5) are sought of the form

\[
a(x, y, t) = a'(t)e^{ikx\sin(y\pi/L_y)},
\]

where \( a \) represents each of the independent variables. Thus all variables are zonally periodic with wavenumber \( kL_x/2\pi \) and are half-sinusoidal meridionally. Under these assumptions
all that is left is to solve for the time-dependent amplitude of each independent variable, that is for $\psi_1(t)$, $\psi_2(t)$, and $T(t)$. In this case the system can be written as

$$\frac{\partial \mathbf{x}}{\partial t} = \mathbf{A}\mathbf{x}, \quad (6)$$

where $\mathbf{x}$ is the state vector $[\psi_1, \psi_2, T]$ and $\mathbf{A}$ is the tangent linear system operator. In order to solve equation (6) and perform linear stability analysis on the system, the operator $\mathbf{A}$ was generated numerically. Calculation of the eigenmodes of this system was then performed with standard numerical techniques on the 3 by 3 matrix $\mathbf{A}$, yielding three damped or growing oscillatory modes. Numerical calculation of the system eigenmodes is equivalent to the analytical method used by QJ97, albeit with all the advantages and disadvantages of the numerical solution. Unless otherwise stated we consider the solution for wavenumber 2, as this is the wavenumber suggested by observations to contain greatest interannual variance in SST. However, wavenumber 3 is also considered as general circulation modelling studies have found this to be the dominant spatial pattern (eg Cai et al 1999, Christoph et al 1998).

### 3 Modes of Variability

As stated above the system (6) has 3 modes. For the above set of parameters two of these are unstable. The structure of the most unstable mode is depicted in Figure 1. It represents a predominantly barotropic ocean oscillation, with streamfunction leading the SST response by approximately 30 days, and SST in turn leading SLP by approximately 15 days. This mode propagates westward, with a phase speed of approximately 134 cm s$^{-1}$ and a period of approximately 90 days. The second unstable mode also propagates westward but much more slowly, with a period of approximately 18 years. This mode
represents a slow oscillation in the lower ocean layer. The third and only damped mode is related to the first. It again represents a barotropic ocean streamfunction response leading SST. In this mode SLP is virtually in-phase with streamfunction and hence also leads SST. This mode propagates eastwards with a phase speed of approximately 15 cm s$^{-1}$, and has a period of approximately 2.4 years. The characteristics of the modes are summarised in Table 1.

The first and third modes of the coupled system are strongly related to the two modes of the uncoupled ocean operator (equations (1)-(2) with curl($\tau$) = 0). The two ocean-only modes have essentially barotropic structure, and their frequencies and propagation characteristics are very similar to those of the first and third modes of the coupled system. This indicates that the first and third coupled modes can be understood as representing the free oscillation of the ocean, with their spatial propagation little affected by the coupling, and a somewhat passive response in SST and SLP. The ocean circulation drives an SST signal that lags streamfunction by a quarter cycle, in phase with the maximum in southwards velocity, which in turn produces an SLP maximum to the east of the SST maximum. For the first, westward propagating coupled mode, the resulting SLP pattern is almost in phase with streamfunction, hence reinforcing the ocean circulation through in-phase wind stress curl forcing. For the third, eastward propagating coupled mode, SLP is almost exactly out of phase with streamfunction and the feedback is negative.

The analysis presented above suggests that the dominant response of the coupled model takes the form of rapidly growing barotropic ocean disturbances with correlated SST and SLP response and period of around 90 days. This is in contrast to the results of QJ97, who discuss only two modes of the system, and found the least stable of these to have a frequency of 4.6 years and to have SLP leading SST. Thus the mode from their analysis was very reminiscent of the ACW, while here none of the three modes of our coupled
system closely resemble either of the two modes from QJ97 or the ACW. The mode that most closely resembles the ACW in phase relationship and period is the only damped mode of the coupled system, and hence the least likely to be observed. The discrepancy between our results and those of QJ97 may originate in the assumptions made in that work in order to reduce the order of the eigenvalue problem to 2. The above analysis was repeated for other wavenumbers and, apart from the expected changes in frequency, the character of the coupled modes was consistent with that discussed above. Owing to the large number of parameters and variables an exhaustive sensitivity study was not performed, but preliminary investigation revealed no ACW-like least damped modes for small changes in any parameters.

The modes of the uncoupled ocean operator have been shown to dominate the coupled system. The fact that the system does not produce an ACW-like response is linked to this dominance. When sufficient damping was added to the otherwise neutral ocean modes an ACW-like response was possible in the coupled system. A linear damping term of the form $-r\psi_i$ was added to the right hand side of equations (1) and (2), and for values of $r > 10^{-5}$ s$^{-1}$ the fastest growing coupled mode has similar frequency and phase relationship to the ACW observations. For such heavy damping, however, the coupling between SST and ocean streamfunction lose significance, rendering the system essentially uncoupled. Thus for the heavy ocean damping the modes of the coupled system tend towards the modes of the individual components of the system, that is to the two ocean modes plus the mode of the SST operator (equation (3) with $\psi_1 = 0$). The latter is referred to hereafter as the SST mode, and it is this mode that resembles the ACW.

From equation (3) it can be seen that the SST mode is damped at the thermal damping rate $\kappa_o$ and has a period of $2\pi/kU_1$. Its propagation is eastward, being advected at the speed of the mean flow, and for the parameters used above has a period of approximately $3$
years. Clearly this mode is more reminiscent of the ACW than those discussed above for the coupled model. For the parameters chosen by Qi97 the period of the SST mode is shorter than that of the ACW, but has the ACW period for another justifiable choice of mean ocean surface velocity $U_1 = 8 \text{ cm s}^{-1}$. Alternatively $U_1 = 5 \text{ cm s}^{-1}$ and wavenumber 3 also yields the observed ACW period. (Coupled model results and the dominant wavenumber 3 pattern in Southern Hemisphere SLP suggest that an ACW of wavenumber 3 is possible.) Thus it appears that the fully coupled model is not necessary for reproducing an ACW-like oscillation in SST, but rather just the SST advection equation (3). Christoph et al (1998) and Weisse et al (1999) found that models similar to (3) could describe to first order the SST variability in their coupled models. Weisse et al (1999) included a linear feedback in their simple model but found this to be of secondary importance. The observations of White and Peterson (1996) also suggest that the ACW is simply advected by the mean flow, consistent with a model such as (3).

4 Stochastically Forced Modes

In the absence of a positive feedback provided by coupling to the ocean and atmosphere, sustenance of a damped mode such as the SST mode is still possible if the mode is continually forced. A plausible candidate to provide such forcing of SST is the (uncoupled and uncorrelated) variability of the atmosphere and ocean, through heat flux variations in the former, and anomalous horizontal and vertical advection of SST in the latter. Compared to the time-scale of the SST mode much of this variability is high frequency and as a result may be considered essentially stochastic in time. It has become common practice for example to treat high frequency wind stress variability as stochastic noise forcing for the more slowly responding ocean (eg Frankignoul and Reynolds 1983). In addition numerous
studies have indicated that coherent variability can result in linear damped geophysical systems that are stochastically forced (eg Farrell and Ioannou 1995, Kleeman and Moore 1997). Christoph et al (1997) and Weisse et al (1999) both demonstrated that temporally stochastic forcing plays a major role in sustaining the interannual SST anomalies in their models. The effect of stochastically forcing the SST equation (3) was investigated also here. In this case the stochastic forcing is chosen to simulate random variations in anomalous meridional ocean surface velocity. This can be thought of as equivalent to replacing the dynamical ocean model with a stochastic term in the coupled system. Although this approach would not be valid in the coupled system above, as the ocean modes are of comparable frequency to the mode of the SST equation, introducing the stochastic forcing is consistent with the real system, where much of the variability in ocean surface velocity exists in mesoscale features with much shorter time-scales than the SST mode.

The stochastically forced system takes the form

$$T(t+1) = T(t) - (\kappa + ikU_1)T(t)\Delta t + \xi(t)T_{\psi},$$

where $\Delta t$ is the time-step, and $\xi$ contains the stochastic forcing. Equation (7) can be recognised as a discretised version of equation (3) with $\psi_1$ replaced by $\xi$. In this case $\Delta t$ is 1 week, and $\xi(t)$ is randomly sampled between -0.05 and 0.05, representing meridional current variations of order 5 cm s$^{-1}$. In equation (7) the forcing is independent of the system state while in the real system it may be expected that the forcing, in this case through mesoscale ocean variability, is state-dependent to some degree (eg Sura 2001, Gent and McWilliams 1990). A time-series of $T$ is shown in Figure 2, along with a plot of the power spectral density, for an intergration of (7) for 1000 years. This reveals oscillations in SST on the order of 0.5$^\circ$C centred on a period of approximately 3 years. As would be anticipated, the dominant response frequency corresponds to the eigenfrequency of the SST operator.
(3). The amplitude of the response is of the same order as that associated with the ACW. Thus it is possible to sustain the damped SST mode at an ACW-like amplitude by forcing with a stochastically varying meridional current of realistic magnitude. The stochastic forcing term may equally represent other forcing mechanisms, including Ekman pumping or atmospheric heat flux.

In the coupled model SLP is a diagnostic, completely determined from SST by an equivalent barotropic response to surface heating. For the SST mode the corresponding solution for SLP is

\[ p(t) = b\lambda\kappa_0/(\lambda T_{ao}/f_0 + \kappa_0) \cdot e^{-(\kappa_0 + i\lambda U_1)t}, \]

which, for the parameters used above, has SLP leading SST by 73°, reminiscent of the 90° phase difference between SST and SLP in the observed ACW. For wavenumber 3 the phase difference is 79°. Thus the stochastically forced system comprising just the SST and SLP equations (equations (3) and (4)) shares a number of the characteristics of the ACW, and appears to be a better model for the ACW than the fully coupled system considered initially.

5 1-Dimensional Model

The model used above assumes homogeneity of the mean circulation and imposes a zonal and meridional structure for the anomalies. To include a greater degree of realism a degree of freedom was added to the model. Rather than allowing only strictly zonal variability we choose the new degree of freedom to be along the axis of the Antarctic Circumpolar Current (ACC), which is a more natural coordinate for advection of anomalies. In other words in this model we consider the evolution of SST anomalies driven by advection of the
background SST field normal to the path of the ACC.

In order to obtain a well resolved and dynamically consistent approximation of Southern Ocean surface velocity and temperature fields, we use data from the OCCAM project, a 12 year simulation of the global ocean circulation using a 1/4 degree primitive equation model (Webb et al 1998). Mean and variability in the surface velocity and temperature fields were calculated from four years of monthly OCCAM data, for the region between 30° S and 70° S. These fields were averaged into bins of size 4° latitude by 8° longitude, yielding data at 10 latitude and 45 longitude bins. The path of the ACC was determined by finding the latitude with largest eastward velocity at each longitude, giving an ACC path length of approximately 27000 km. The mean along-path velocity was approximately 10 cm s\(^{-1}\), with a maximum of approximately 12 cm s\(^{-1}\) at 140° W and minimum of approximately 6 cm s\(^{-1}\) at 124° E, giving an advection time-scale of approximately 8.5 years. The tangent linear system operator was determined from:

\[
B = -U_p D_p - \kappa_o,
\]

where \(p\) is the along-path coordinate, \(U_p\) contains the along-path velocity, \(D_p\) is the finite difference along-path derivative operator, and \(T_t = BT\). In this case \(T\) is a length 45 vector containing the SST anomaly at each bin along the ACC path, and \(B\) is a 45 by 45 matrix.

This system was stochastically forced as above, although on this occasion both the spatial and temporal structure are stochastic, whereas for the earlier model the forcing by construction had fixed zonal wavenumber. An ensemble of one hundred 100 year simulations were run, with a time-step of 1 week. The magnitude of the forcing was initially set to be \(< T_n^* > < u_n >\), where \(< . >\) signifies the along-path average, the subscript \(n\) signifies the derivative normal to the ACC path, \(T^*\) is the mean SST as above, and \(u\)
is the standard deviation of anomalous sea surface velocity. In this case $T^*$ and $u$ were determined from the OCCAM data, yielding $< T^*_n > \approx 0.8 \, ^\circ C \, (\text{lat.})^{-1}$ and $< u_n > \approx 8 \, \text{cm s}^{-1}$. For the present case the forcing is isotropic - the effect of using a more realistic spatial structure for the forcing is considered below. Note that the forcing may equally be interpreted as a stochastic vertical heat flux due to Ekman pumping or atmospheric heat flux of appropriate magnitude. Southern Ocean surface temperature variability due to vertical flux processes was of the same order as that due to horizontal advection in the coupled GCM analysed by Rintoul and England (2002). The forcing described above would equate to an anomalous wind stress curl of order $10^{-7} \, \text{N m}^{-2}$ coupled to a vertical temperature gradient of order $10^{-2} \, ^\circ C \, \text{m}^{-1}$, and for the latter case to an anomalous heat flux of order $20 \, \text{W m}^{-2}$, each of which may be considered of reasonable order for the Southern Ocean.

A 50 year segment of the system response to the stochastic forcing is presented in the Hovmoeller diagram of Figure 3a. The data in this figure have been low-pass filtered to remove disturbances with time-scales less than 1 year. The figure suggests the presence of large-scale eastward propagating anomalies. While most individual anomalies are not truly circumpolar, many can be seen to persist for over 180 degrees of longitude. The dominance of large-scale low-frequency disturbances in the model response is confirmed in Figure 4, which shows the power spectral density as a function of along-path wavenumber and of frequency for the unfiltered ensemble simulations. Despite the fact that the forcing is spatially and temporally uncorrelated, the system variability resides predominantly in low wavenumbers and in frequencies $< (2 \, \text{years})^{-1}$. The amplitude of the interannual SST anomalies is up to $1 \, ^\circ C$ - of the same order as those associated with the ACW from observations.

As was the case above, the system response may be explained from eigenanalysis of the linear operator $B$. Details of the least damped modes of $B$ are given in Table 2. The least
damped mode simply corresponds to the uniform decay of all anomalies at the thermal damping rate \( \kappa_0 \). The next least damped modes correspond to anomalies of increasing along-path wavenumber and decreasing frequency. For the modes of low wavenumber the decay rates increase only gradually with wavenumber, while the frequencies of the first four wavenumbers are \( < (2 \text{ years})^{-1} \). From this it may be seen that the system response to stochastic forcing corresponds well to the least damped modes in both wavenumber and frequency. The stochastic forcing excites all modes, but variance is maintained predominantly in the interannual, low wavenumber modes, as these are the least damped.

Thus in this simple model coherent low wavenumber interannual SST anomalies were sustained by a forcing that was random in both space and time. The model spectra differ from that of the observed ACW in that no dominant wavenumber or frequency was seen - a result of the relatively small differences in the decay rates of low wavenumber modes and the fact that in the ensemble mean they were evenly excited by the white noise forcing. Note, however, that on shorter time-scales (eg in some individual 100 year simulations) dominant spectral peaks do appear at the low wavenumbers and interannual frequencies, and that it is only in the ensemble mean that the broadness of the spectrum is assured. This suggests the possibility that the 13 years of SST data available for the analysis of White and Peterson (1996), despite appearing dominantly unimodal over this time period, may be in fact representative of a broad spectrum stochastically forced system such as the model studied here. That is, the apparent unimodal nature of the SST variability in the ACW observations may be an artifact of the shortness of the record. Christoph et al (1998) note similarly; around 20% of their GCM simulations had a wavenumber 2 structure consistent with the ACW observations, but in the ensemble average wavenumber 3 was equally important. Application of a 3 to 7 year filter as was employed by White and Peterson (1996) increases the unimodal appearance of the data by removing much of the
variability from the higher wavenumbers.

The forcing due to anomalous cross-path advection of the mean SST field considered thus far is of order $10^{-7} \, ^\circ C \, s^{-1}$. As mentioned above such a forcing could equally be interpreted as being a stochastic vertical heat flux due to Ekman pumping or atmospheric heat flux variations of reasonable order for the Southern Ocean. That is, stochastic isotropic forcing from any of these sources is capable of sustaining the model response presented above. Differences in the model response arise only when account is taken of the differing (non-isotropic) spatial structure of each forcing. The OCCAM data suggest that anomalous cross-path SST advection is dominated by forcing south of Africa, where strong meridional SST gradients coincide with robust mesoscale eddy activity. Thus a zonally varying forcing based on the local mean SST gradient and anomalous cross-path velocity variability produces maximum anomalous SST south of Africa at the site of greatest forcing, gradually reducing in magnitude across the Pacific sector before increasing again east of Drake Passage. A 50 year segment of the model response to such a forcing is shown in Figure 3b.

The well-documented dominant standing wavenumber 3 pattern of anomalous SLP over many time-scales in the Southern Hemisphere (Mo and White, 1985) suggests that anomalous atmospheric heat flux, Ekman pumping and cross-path Ekman transport may each be expected to contain a significant wavenumber 3 component. Forcing the model with a stochastically varying fixed phase wavenumber 3 pattern, with amplitude corresponding to a vertical heat flux of order 20 W m$^{-2}$, Ekman pumping of order $10^{-5}$ m s$^{-1}$, or Ekman transport of order 5 cm s$^{-1}$, sustains a wavenumber 3 response in SST of order 1$^\circ C$ (Figure 3c). The GCM studies of Christoph et al (1998), Weisse et al (1999) and Bonekamp (1999) find vertical heat flux to be the primary source of ACW-like SST anomalies, while the coupled climate model analysis of Rintoul and England (2002) suggests Subantarctic
Mode Water temperature variability to be largely driven by Ekman transport across the path of the ACC.

These results demonstrate that, while for white-noise isotropic forcing dominance of one mode vanishes in the ensemble mean, long-term dominance of one mode is possible when the forcing consistently projects more strongly onto that mode. Alternatively a dominant wavenumber 3 response can be achieved through an isotropic forcing that is harmonic at the resonant frequency of the wavenumber 3 SST mode. The response of a simplified model from Weisse et al (1999) similar to that used here may be understood as the atmospheric forcing in their model preferentially exciting the wavenumber 3 mode of the SST advection operator in their system.

6 Conclusion

A simple linearised coupled model of the Southern Ocean and overlying atmosphere has been examined for modes of variability reminiscent of the ACW. Contrary to previous analysis of the model, no such modes were found, self-sustaining or damped. The most unstable coupled mode was dominated by the free oscillation of a westward propagating ocean mode driving a relatively rapid SST response. The mode of the SST component of the coupled system does, however, resemble the ACW in a number of aspects, and as a result when the ocean modes were heavily damped in the coupled system, essentially equivalent to decoupling the model, the SST response of the system was ACW-like. This suggests that explicit coupling from SST to ocean velocity (via SLP) is not necessary in order to produce an ACW-like response in the system. This lends support to previous studies that have been able to understand much of the interannual Southern Ocean SST variability in coupled models using uncoupled advection equations similar to equation (3). In this study
we have demonstrated that such a mode may be sustained at an amplitude comparable to the ACW through stochastically varying meridional ocean surface velocity fluctuations of order 5 cm s$^{-1}$. It was found that SLP anomalies driven through a barotropic response to SST have a similar phase relationship to the ACW. These conclusions were found to be qualitatively robust to the choice of wavenumber. In particular for wavenumbers 2 and 3, the wavenumbers inferred from observations and models respectively, modes with ACW-like characteristics were possible for different justifiable choices of mean surface velocity. It is concluded then that the ACW may be explained as a stochastically forced mode of the SST field that forces a correlated response in SLP, but without any significant feedback.

Upon extending the model to allow variability along the path of the ACC, the least damped of the system eigenmodes were found to be low wavenumber and interannual. Due to the decay rate increasing only gradually with wavenumber for small wavenumbers, a long stochastically-forced simulation of the system demonstrated variance spread among low wavenumbers and interannual frequencies. Such a broad spread of spectral power is not seen in the ACW observations, however, shorter segments sampled from simulations using our model can resemble the ACW SST spectra, suggesting the possibility that the ACW observations may be consistent with a short sampling from a broadband stochastically forced system. Anisotropic forcing of this model was also able to sustain a dominant wavenumber/frequency through preferential excitement of one mode; excitation of the wavenumber 3 mode for instance was demonstrated by forcing the model either with a stochastically varying random phase wavenumber 3 pattern or with isotropic harmonic forcing near the mode’s resonant frequency. Such a wavenumber 3 forcing could be derived from vertical or horizontal heat flux processes linked to the dominant wavenumber 3 pattern of Southern Hemisphere SLP. As discussed above, our stochastic forcing through cross-path (meridional) advection of mean SST can be regarded as an equivalent vertical heat flux.
In either context, we have demonstrated that stochastic forcing of the mean SST field is of sufficient magnitude in the Southern Ocean to induce anomalous upper ocean temperature fluctuations of order 1°C, comparable to those seen in the ACW observations.

Acknowledgements. This research was supported by the Australian Research Council.
References


<table>
<thead>
<tr>
<th>mode number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>QJ97</th>
</tr>
</thead>
<tbody>
<tr>
<td>period</td>
<td>98.7 days</td>
<td>17.9 years</td>
<td>2.4 years</td>
<td>4.6 years</td>
</tr>
<tr>
<td>growth rate</td>
<td>0.01 years$^{-1}$</td>
<td>48.8 years$^{-1}$</td>
<td>-0.01 years$^{-1}$</td>
<td>0.09 years$^{-1}$</td>
</tr>
<tr>
<td>propagation direction</td>
<td>westwards</td>
<td>westwards</td>
<td>eastwards</td>
<td>eastwards</td>
</tr>
<tr>
<td>SLP leads SST?</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 1: Summary of characteristics of modes of the simple coupled model. The fourth column gives the values for the most unstable mode from the analysis of QJ97. Negative growth rates indicate decay.
<table>
<thead>
<tr>
<th>mode number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>wavenumber</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>period</td>
<td>-</td>
<td>8.6 years</td>
<td>4.3 years</td>
<td>2.9 years</td>
</tr>
<tr>
<td>decay time-scale</td>
<td>1 year</td>
<td>0.96 years</td>
<td>0.84 years</td>
<td>0.70 years</td>
</tr>
</tbody>
</table>

Table 2: Summary of characteristics of least damped modes of the 1-dimensional SST advection operator.
Figure 1: The structure of the least damped mode of the coupled model, using the parameters from QJ97. Note that the mode propagates westwards, so that SST leads SLP in this mode. The amplitudes of each variable have been normalised.
Figure 2: The top two panels show a segment from the stochastically forced SST equation and the corresponding response in SLP. The bottom two panels show the power spectral density (psd) for SST and SLP in units of \((^\circ C)^2/\text{cpy}\) and \(\text{Pa}^2/\text{cpy}\) respectively (cpy=cycles per year).
Figure 3: Hovmoeller diagram of the response of the 1-dimensional SST advection model to stochastic forcing. Shown is the response to a) isotropic forcing, b) forcing proportional to the local SST gradient and cross-path velocity variability, and c) wavenumber 3 forcing. Data in each panel has been low-pass filtered to remove variability from periods less than 1 year.
Figure 4: Power spectral density of stochastically forced SST as a function of along-path wavenumber (top) and frequency (bottom), averaged over an ensemble of 100 simulations of 100 years each. Units of power spectral density are (°C)²/cpy (cpy=cycles per year).