List of Corrections

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This file contains comments and corrections to the book

> J. Dick and F. Pillichshammer, Digital Nets and Sequences. Discrepancy Theory and quasi-Monte Carlo Integration. Cambridge University Press, Cambridge, 2010.

For a preprint version see http://web.maths.unsw.edu.au/~josefdick/preprints/DP_book_preprint.pdf. Note that the preprint version differs from the published book. In particular, the page numbers are different. However, numbers of Chapters, Sections, Theorems, Lemmas, Corollaries, Definitions and Examples are the same.

Please let me know if you know of any further mistakes or have other constructive comments.

Bibtex

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@book {DP10,
AUTHOR = {Dick, Josef and Pillichshammer, Friedrich},
TITLE = {Digital nets and sequences. Discrepancy theory and
         quasi-Monte Carlo integration},
PUBLISHER = {Cambridge University Press, Cambridge},
YEAR = {2010},
PAGES = {xviii+600},
ISBN = {978-0-521-19159-3},
MRCLASS = {65C05 (11K06 11K38 11K45 46E35 65D30)},
MRNUMBER = {2683394 (2012b:65005)},
DOI = {10.1017/CBO9780511761188},
URL = {http://dx.doi.org/10.1017/CBO9780511761188},
}
```

Corrections

We gratefully acknowledge the help of several colleagues for pointing out typos in our book: These are Michael Feischl, Daniel Gerigk, Michael Gnewuch, Takashi Goda, Andreas Griewank, Stephen Joe, Peter Kritzer, Hernan Leovey, Gottlieb Pirsic, Ian Sloan, Takehito Yoshiki.

In the following list, line n means n-th line from the top and line $-n$ means n-th line from the bottom.

Page XII, line 7: "have" should be replaced by "has".

Page 8, line 6: $\int_{[0,1]^s} f(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}$ should read $\int_{[0,1]^s} g(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}$.

Page 9, line 11: Replace $[0,1]^s$ in the definition of W_n by $\mathbb{R}^s$.

Page 15, Exercise 1.10: The function f must be continuous and monotone.

Page 28, Proposition 2.11: $\mathcal{P}$ should be replaced by 0 in the formula for the initial error.

Page 42, Exercise 2.3: The inner product should be defined as $\langle f, g\rangle = a_0\overline{b_0} + a_1\overline{b_1} + \cdots + a_r\overline{b_r}$.

Page 68, line 13: The result Niederreiter [177, Theorem 3.10] was first shown 15 years earlier as Lemma 2.2 in the paper H. Niederreiter, Pseudo-random numbers and optimal coefficients, Advances in Math. 26, 99 – 181 (1977).

Page 68, line -7: replace "[175, Chapter 3]" by "[177, Chapter 3]".

Page 85, line 6: Replace "Section 5" by "Chapter 5".

Page 87, line -3: Replace "Section 5" by "Chapter 5".

Page 88, line 2: Replace "Section 5" by "Chapter 5".

Page 109, Definition 4.1 and 4.2: The sets $\mathcal{P}$ are multisets.

Page 146, line 3: Replace "Section 4" by "Chapter 4".

Page 151, Eq. (4.8): line 4: $e_2^{(1)}$ should be $e_1^{(2)}$ and in line 6: $e_{d_2}^{(1)}$ should be $e_{d_2}^{(2)}$.

Page 180, paragraph 2, line 2: Replace "Niederreiter [168]" by "Niederreiter [172]".

Page 192, line -11: write $1 \leq c \leq c_m$ instead of $0 \leq c \leq c_m - 1$.

Page 198, Theorem 5.28: replace "then" by "than".

Page 214, last line: The sum is over all $(C_1, \ldots, C_s)$ instead of C.

Page 234, paragraph 1, line 3: Replace "[170]" by "[172]".

Page 256, line -11: "Theorem 7.1" should read "Theorem 14".

Page 260, line -12: replace $C_i^{(m+1)}$ by $C_j^{(m+1)}$.

Page 308, line 11: The factor b^d is missing.

Page 311, line 2: It should read "Lemma 5.35" instead of "Lemma 8.8".

Page 325, line -3: It should read $C = (\Pi(g))^\top \omega_p \Pi(g^{-1})$.

Page 326, line -5: Replace $(c_0, c_{n-1}, c_{n-2}, \ldots, c_1)^\top$ by $(c_0, c_1, c_2, \ldots, c_{n-1})^\top$.

Page 327, lines 8, 9, 12: Replace $\Pi(g)^\top$ by $\Pi(g)$.

Page 327, line 11: Replace $\mathbb{C}^n$ by $\mathbb{C}^{b^m-1}$.

Page 327, line 12: Replace $\mathcal{O}(n)$ by $\mathcal{O}(b^m)$.

Page 327, lines 13, 14: Replace $\mathcal{O}(n \log n)$ by $\mathcal{O}(mb^m)$.

Page 365, line -8: There is an inaccuracy in the proof of Lemma 12.2: $|\boldsymbol{x} - \boldsymbol{y}|_\infty < \delta$ does not necessarily imply that $\xi_{i,k} = \eta_{i,k}$ for all $1 \leq i \leq s$ and $1 \leq k \leq a$. For example, in the case $s = 1$, $b = 2$ and $x = 0.1$ we can choose $y = 0.011\ldots10\ldots$ arbitrarily close to x, but the first dyadic-digits of x and y are always different. Nevertheless, Lemma 12.2 remains valid and we give a correct proof in the Appendix.

Page 367, line 4: It should read "Proposition 2.11" instead of (2.11).

Page 368, Equation (12.1): A parenthesis ")" is missing at the end of the equation.

Page 396, line -2: Period is missing after $\cdots$.

Page 404, Equation (13.10): Replace ${}_b\mathrm{wal}_{\kappa'_{j-1}}(\eta'_j/b)$ by $\overline{{}_b\mathrm{wal}_{\kappa'_{j-1}}(\eta'_j/b)}$.

Page 404, line -5: The summation in the second sum should be over η_{r+1} instead of y_{r+1}.

Page 408, line -8: The gain coefficients are $\Gamma_{\boldsymbol{\ell}} := NG_{\boldsymbol{\ell}}$.

Page 412, line 19: ...smaller than b^s times the variance for a (plain) MC algorithm.

Page 444, line 1: replace "Sterling" by "Stirling".

Page 462, line 14: Instead of $\prod_{i=1}^{s}$ it should read $\prod_{i \in \mathfrak{u}}$.

Page 463, line 3: delete $\gamma_{\mathfrak{u}}^{-1}$.

Page 465, line -7: Instead of ${}_b\mathrm{wal}_{\boldsymbol{k}}(\boldsymbol{x})$ write ${}_b\mathrm{wal}_{\boldsymbol{k}}(\boldsymbol{x}_n)$.

Page 466, line 4: Replace $b^{-\mu_\alpha}(\boldsymbol{k})$ by $b^{-\mu_\alpha(\boldsymbol{k})}$.

Page 466, line 7: Replace $a_1 > a_2 > \cdots > a_\nu \geq 1$ by $d_1 > d_2 > \cdots > d_\nu \geq 1$.

Page 466, line 10: Write $\mu_\alpha(\boldsymbol{k})$ instead of $\mu_{\boldsymbol{\alpha}}(\boldsymbol{k})$ (i.e., α should not be bold).

Page 475, line -17: Replace the j by i (four times).

Page 488ff, Lemma 15.20, Theorem 15.21, Corollary 15.22: Here we need to assume that $\alpha \geq 2$, as stated at the beginning of Section 15.6.

Page 497, line 11: It should read $l - \alpha d_\alpha$ instead of $l - \alpha a_\alpha$.

Page 510, line 8: Replace "Theorem 16.1" by "Theorem 3.20".

Page 591, ref. [171]: the year of publication is 1978.

Comments

In the following we comment on parts of the book which might be confusing.

Page 22, line 12, 13: It is not necessary to switch from $K(\cdot, x_n)$ to $K(x_n, \cdot)$ and analogously for $K(\cdot, x_m)$.

Page 95, line 5: The usage of the weights for different q-norms is inconsistent. To make the weights consistent with the classical usage (as in Sloan & Woźniakowski, J. Complexity 14 (1998), no. 1, 133) in the 2-norm, then the weight $\gamma_{\mathfrak{u},s}$ should be replaced by $\gamma_{\mathfrak{u},s}^{q/2}$. However, if one wants to make the weights consistent with the ∞-norm in line 3 on the same page, then one should replace the weight $\gamma_{\mathfrak{u},s}$ by $\gamma_{\mathfrak{u},s}^{q}$.

Page 210, line 2: The inequality sign $\leq$ can be replaced by an equality sign $=$.

Appendix

Correction of the proof of Lemma 12.2

We restrict ourselves to the case $s = 1$. For $x = \xi_1 b^{-1} + \xi_2 b^{-2} + \cdots$ and $\sigma = \varsigma_1 b^{-1} + \varsigma_2 b^{-2} + \cdots$ let

$$x \oplus \sigma = (\xi_1 \oplus \varsigma_1) b^{-1} + (\xi_2 \oplus \varsigma_2) b^{-2} + \cdots ,$$

where for $\xi, \varsigma \in \{0, 1, \ldots, b-1\}$ we define $\xi \oplus \varsigma = x + \varsigma \pmod b$.

Let $\mathbb{Q}(b^l) = \{k/b^l \ : \ 0 \leq k < b^l\}$ and $\mathbb{Q}(b^\infty) = \bigcup_{l=0}^{\infty} \mathbb{Q}(b^l)$. The elements from $\mathbb{Q}(b^\infty)$ are called the b-adic rational points of $[0,1)$. Note that $\lambda(\mathbb{Q}(b^\infty)) = 0$.

We show a property of the digital shift:

Lemma 1 *Let $\sigma = \varsigma_1 b^{-1} + \varsigma_2 b^{-2} + \cdots \in [0,1)$. The mapping $x \mapsto x \oplus \sigma$ is continuous in every $x \in [0,1) \setminus \mathbb{Q}(b^\infty)$ and right-continuous in every $x \in \mathbb{Q}(b^\infty)$, uniformly in σ.*

Proof. 1. Let $x \in [0,1) \setminus \mathbb{Q}(b^\infty)$. Assume that $|x - y| < \delta$ for some $0 < \delta \leq b^{-1}$. Let $a \in \mathbb{N}$ be such that $b^{-a-1} < \delta \leq b^{-a}$. Then it follows that $\xi_k = \eta_k$ for all $1 \leq k \leq a$, where η_k are the b-adic digits of y. Hence $\xi_k \oplus \varsigma_k = \eta_k \oplus \varsigma_k$ for all $1 \leq k \leq a$ and therefore $|(x \oplus \sigma) - (y \oplus \sigma)| \leq b^{-a} < b\delta$.

2. Let $x \in \mathbb{Q}(b^l)$ for some $l \in \mathbb{N}_0$. Then $x = \xi_1 b^{-1} + \xi_2 b^{-2} + \cdots + \xi_l b^{-l}$. Let $y \geq x$ (which implies that $\xi_k \leq \eta_k$ for all $k \in \mathbb{N}_0$, where we also use the finite expansion if $y = x$) such that $|x - y| < \delta \leq b^{-a}$. Then we have again that $\xi_k = \eta_k$ for all $1 \leq k \leq a$. (Note that this is not true for $y < x$. For example, in the case $b = 2$ and $x = 0.1$ we can choose $y = 0.011\ldots10\ldots$ arbitrarily close to x, but the first dyadic-digits of x and y are always different.) The rest of the proof is like in the case above.

□

We also need a version of Theorem A.20 with less demanding conditions:

Lemma 2 *Let $f : [0,1]^2 \to \mathbb{R}$ be such that*

1. *$f(x,y)$ is continuous in every (x,y) for which $x,y \in [0,1) \setminus \mathbb{Q}(b^\infty)$;*
2. *$f(x,y)$ is continuous with respect to the first and right-continuous with respect to the second variable whenever $x \in [0,1) \setminus \mathbb{Q}(b^\infty)$ and $y \in \mathbb{Q}(b^\infty)$;*
3. *$f(x,y)$ is right-continuous with respect to the first and continuous with respect to the second variable whenever $x \in \mathbb{Q}(b^\infty)$ and $y \in [0,1) \setminus \mathbb{Q}(b^\infty)$;*
4. *$f(x,y)$ is right-continuous in every (x,y) for which $x,y \in \mathbb{Q}(b^\infty)$.*

Assume further that $\sum_{k,l=0}^\infty |\widehat{f}(k,l)| < \infty$. Then $\sum_{k,l=0}^\infty \widehat{f}(k,l)\, {}_b\mathrm{wal}_{(k,l)}(x,y)$ converges uniformly to $f(x,y)$ and we have

$$f(x,y) = \sum_{k,l=0}^\infty \widehat{f}(k,l)\, {}_b\mathrm{wal}_{(k,l)}(x,y) \quad \textit{for all } x,y \in [0,1).$$

Proof. For given $x,y \in [0,1)$ we have

$$\begin{aligned}
&\sum_{k=0}^{b^u-1} \sum_{l=0}^{b^v-1} \widehat{f}(k,l)\, {}_b\mathrm{wal}_{(k,l)}(x,y) \\
&= \int_0^1 \int_0^1 f(t,s) \sum_{k=0}^{b^u-1} \sum_{l=0}^{b^v-1} {}_b\mathrm{wal}_k(x) \overline{{}_b\mathrm{wal}_k(t)}\, {}_b\mathrm{wal}_l(y) \overline{{}_b\mathrm{wal}_l(s)} \,\mathrm{d}t \,\mathrm{d}s \\
&= b^{u+v} \int_{b^{-u}\lfloor b^u x \rfloor}^{b^{-u}\lfloor b^u x \rfloor + b^{-u}} \int_{b^{-v}\lfloor b^v y \rfloor}^{b^{-v}\lfloor b^v y \rfloor + b^{-v}} f(t,s) \,\mathrm{d}t \,\mathrm{d}s.
\end{aligned}$$

If $x \in \mathbb{Q}(b^m)$, then $b^{-u}\lfloor b^u x \rfloor = x$ for all $u \geq m$. Hence, for all $x,y \in [0,1)$ the above expression tends to $f(x,y)$ for increasing u and v.

Since $\sum_{k,l=0}^\infty |\widehat{f}(k,l)| < \infty$ it follows that the partial sums

$$\sum_{k=0}^{M} \sum_{l=0}^{N} \widehat{f}(k,l)\, {}_b\mathrm{wal}_{(k,l)}(x,y)$$

are a Cauchy sequence and hence they are convergent with limit $f(x,y)$. □

Now we can prove Lemma 12.2 (for dimension $s = 1$):

Proposition 1 (Page 364, Lemma 12.2) *Let the reproducing kernel $K \in L_2([0,1]^2)$ be continuous and for $k \in \mathbb{N}_0$ let*

$$\widehat{K}(k,k) = \int_0^1 \int_0^1 K(x,y)\, \overline{{}_b\mathrm{wal}_k(x)}\, {}_b\mathrm{wal}_k(y) \,\mathrm{d}x \,\mathrm{d}y.$$

If

$$\sum_{k=0}^{\infty} |\widehat{K}(k,k)| < \infty,$$

then the digital shift invariant kernel K_{ds} is given by

$$K_{\mathrm{ds}}(x,y) = \sum_{k=0}^{\infty} \widehat{K}(k,k)\, {}_b\mathrm{wal}_k(x)\, \overline{{}_b\mathrm{wal}_k(y)}.$$

Proof. In the same way as shown on page 365, lines 1–12, we can prove that

$$\widehat{K}_{\mathrm{ds}}(k,k') = \begin{cases} \widehat{K}(k,k) & \text{if } k = k', \\ 0 & \text{if } k \neq k'. \end{cases}$$

We show now that $K_{\mathrm{ds}}(x,y)$ is continuous or right-continuous depending whether x,y are b-adic rationals or not.

- Let $x,y \in [0,1) \setminus \mathbb{Q}(b^\infty)$: Let $\varepsilon > 0$. Since K is uniformly continuous there exists a $\delta > 0$ which is independent of (x,y) such that for all (x',y') with $\|(x,y)-(x',y')\|_\infty < \delta$ we have

  $$|K(x,y) - K(x',y')| < \varepsilon.$$

  But then, according to Lemma 1 and its proof, we have

  $$|K(x \oplus \sigma, y \oplus \sigma) - K(x' \oplus \sigma, y' \oplus \sigma)| < \varepsilon$$

  for all (x',y') with $\|(x,y)-(x',y')\|_\infty < \delta b^{-1}$ and this holds independently of σ. Hence we have

  $$\begin{aligned} &|K_{\mathrm{ds}}(x,y) - K_{\mathrm{ds}}(x',y')| \\ &\quad \leq \int_0^1 |K(x \oplus \sigma, y \oplus \sigma) - K(x' \oplus \sigma, y' \oplus \sigma)| \,\mathrm{d}\sigma \\ &\quad < \int_0^1 \varepsilon \,\mathrm{d}\sigma = \varepsilon, \end{aligned}$$

  for all $x',y' \in [0,1)^s$ such that $\|(x,y)-(x',y')\|_\infty < \delta b^{-1}$. Therefore K_{ds} is continuous in (x,y).

- Let $x \in \mathbb{Q}(b^\infty)$ and $y \in [0,1) \setminus \mathbb{Q}(b^\infty)$: Let $\varepsilon > 0$. Since K is uniformly continuous there exists a $\delta > 0$ which is independent of (x,y) such that for all (x',y') with $\|(x,y)-(x',y')\|_\infty < \delta$ we have

$$|K(x,y) - K(x',y')| < \varepsilon.$$

But then, according to Lemma 1 and its proof, we have

$$|K(x \oplus \sigma, y \oplus \sigma) - K(x' \oplus \sigma, y' \oplus \sigma)| < \varepsilon$$

for all (x',y') with $x' \geq x$ and $\|(x,y)-(x',y')\|_\infty < \delta b^{-1}$ and this holds independently of σ. Hence we have

$$\begin{aligned} & |K_{\mathrm{ds}}(x,y) - K_{\mathrm{ds}}(x',y')| \\ & \quad \leq \int_0^1 |K(x \oplus \sigma, y \oplus \sigma) - K(x' \oplus \sigma, y' \oplus \sigma)| \,\mathrm{d}\sigma \\ & \quad < \int_0^1 \varepsilon \,\mathrm{d}\sigma = \varepsilon, \end{aligned}$$

for all $x', y' \in [0,1)$ such that $x' \geq x$ and $\|(x,y)-(x',y')\|_\infty < \delta b^{-1}$. Therefore $K_{\mathrm{ds}}(x,y)$ is right-continuous with respect to the first and continuous with respect to the second variable.

- The remaining cases can be shown by a similar reasoning.

Under the assumption that $\sum_{k=0}^\infty |\widehat{K}(k,k)| < \infty$ we obtain from Lemma 2 that

$$K_{\mathrm{ds}}(x,y) = \sum_{k=0}^\infty \widehat{K}(k,k) \,{}_b\mathrm{wal}_k(x) \overline{{}_b\mathrm{wal}_k(y)}.$$

□