Critical Studies / Book Reviews

James Franklin. ***An Aristotelian Realist Philosophy of Mathematics.*** New York, Palgrave Macmillan, 2014. ISBN: 978-1-137-40072-7 (hbk); 978-1-137-40073-4 (pdf); 978-1-137-40074-1 (e-book). Pp. x + 308.

Reviewed by **Max Jones***

Most of the traditional problems in the philosophy of mathematics arise, in James Franklin's words, out of the 'oscillation between Platonism and nominalism, as if those were the only alternatives' (p. 11). In *An Aristotelian Realist Philosophy of Mathematics* Franklin develops a tantalizing alternative to these approaches by arguing that at least some mathematical universals exist in the physical realm and are knowable through ordinary methods of access to physical reality. By offering a third option that lies between these extreme all-or-nothing approaches and by rejecting the 'dichotomy of objects into abstract and concrete', Franklin provides potential solutions to many of these traditional problems and opens up a whole new terrain for debate in the philosophy of mathematics (p 15). The acknowledgement of this by no means new but oft neglected Aristotelian position sheds refreshing new light on debates that have become somewhat stagnant in recent times. Furthermore, by drawing attention to the possibility of an Aristotelian alternative, Franklin opens the way for a whole host of new debates to emerge regarding the correct Aristotelian approach. The scope of the book is ambitious and the overall position defended is controversial in a number of ways. As such, it gives rise to as many new questions as it provides answers. However, this should be seen as a positive, since the many questions that arise are deeply significant and have been neglected by philosophers of mathematics for far too long.

One of the beauties of the Aristotelian position introduced is that it has far more scope for internal variation than either Platonist or nominalist alternatives. Once one takes the step of acknowledging that some mathematical objects reside in our universe, this opens up a whole new range of debates regarding how much of mathematics can be understood in physical terms and in what way. Even if Franklin's is not the ultimate Aristotelian approach, it is a pioneering step that could lead to many more interesting developments. Furthermore, by altering the terms of debate, Franklin paves the way for novel anti-realist alternatives that follow his lead by taking actual mathematical practice and empirical research into the study of mathematical cognition more seriously.

The author's wide-ranging understanding of the actual practice of mathematics stands out throughout and it is refreshing to read a book on the philosophy of mathematics by a practising mathematician. This fresh perspective allows the author to develop a truly novel approach, with much room for further expansion and refinement, without being burdened by some of the stale

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debates that constrain more orthodox philosophers of mathematics. The book is particularly readable, with the author using delightful metaphors throughout to bring colour to what can often be a rather dry topic. Although the author uses examples from a wide range of different mathematical applications, these are all made clear by the author's excellent explanations, and philosophical positions are all introduced without assuming too much background knowledge. The book should be accessible to most mathematicians and philosophers.

The book opens by providing a general account of Aristotelian realism (Ch. 1), emphasising the unwarranted absence of this position in the philosophy of mathematics, given its status as a live option in other areas of metaphysical debate. Through commitment to physically instantiated universals, the Aristotelian approach is able to account for structure, in terms of relational universals and sortal properties, both of which figure heavily in the account of mathematical properties that the author goes on to provide in chapters three and four. Attention is also drawn to the significant fact that Aristotelian accounts are able to offer a perceptual account of our access to knowledge of universals.

The second chapter addresses the central problem for any Aristotelian account that lies in explaining uninstantiated universals. Even if one admits the physical reality of some mathematical objects, most will agree that not all mathematical objects are physically instantiated. The onus therefore is on the Aristotelian to provide some account of those mathematical objects that transcend the physical and of how knowledge of such objects is possible. Franklin adopts a position that he calls semi-Platonist or modal-Aristotelian, according to which mathematics deals with possible physical structures, some of which are actually instantiated. Franklin makes the interesting comparison between uninstantiated mathematical properties and Hume's famous example of the missing shade of blue (p. 23), and he returns to this intriguing comparison between number and colour several times throughout the book. The comparison is particularly powerful, since most would agree that, if there were such a missing shade, it would fall within the remit of colour science despite its mere possibility. However, the fact that one of the objects of such an imagined colour science is a mere possibility does nothing to invalidate the idea that colour science is primarily concerned with features of the actual world. Similarly, the fact that mathematics is sometimes concerned with uninstantiated universals is entirely consistent with the idea that it is primarily concerned with physical reality.

In chapter three, Franklin distinguishes his own position from other supposedly Aristotelian positions, such as Resnik's [1997], which treat all of mathematics as the science of structure. Franklin argues, instead, that elementary mathematics is the 'science of quantity' (p. 31). This move is significant, as it allows Franklin to provide a more realistic picture of the kind of everyday mathematics in which most non-mathematicians engage. Quantity and structure are taken to be sufficiently distinct so as to merit separate accounts and yet closely related enough not to threaten a unified account of mathematical subject matter. One of the problems that has faced previous attempts to understand mathematics as the science of structure is a lack of clarity in defining 'structure', such that structures are the kinds of things that could be instantiated in physical reality.

In chapter four, Franklin provides new insight by offering a novel definition of 'purely structural' properties as those that 'can be defined wholly in terms of the concepts *same* and *different*, and *part* and *whole* (along with purely logical concepts)' (p. 57, emphasis added). He provides some interesting examples of cases where group-theoretic structures are realised in physical reality. For instance, 'the cyclic group of order 2, is a universal literally realised in the Caps Lock toggle on a keyboard' and 'the essence of the abstract structure SO(3), is literally realised in physical rotations' (p. 53). Franklin's insistence on the physical reality of mathematical structure is a bold step and it would be interesting to investigate the impact of such an approach on wider issues in metaphysics and the philosophy of science. For instance, this approach might have interesting implications for certain structural-realist positions that also take structures to be physically instantiated but tend to take these structures to be distinct from mathematical structures.[1]

Chapter five addresses the apparent tension between mathematical truths being necessary and their being truths about reality. At face value, many of the so-called necessary truths about mathematics seem to be about entities that do not exist in reality, such as Euclidean planes and exact regular pentagons. On the other hand, we tend to take most of the truths about the actual universe to be merely contingent. However, Franklin makes the novel point that the existence of necessary truths about entities such as Euclidean planes and exact regular pentagons which do not exist in reality, implies the existence of necessary truths about approximately Euclidean planes and approximately regular pentagons (pp. 67–69). He then goes on to defend his position against a number of objections in a thoroughly convincing manner.

Chapter six highlights the impoverished scope of traditional philosophy of mathematics regarding the mathematical practice that it addresses. The majority of philosophers of mathematics are accused of sticking to the 'traditional diet' of 'numbers, sets, infinite cardinals, axioms [and] theorems of logic' and in doing so neglecting important insights both from the vast swathe of pure and applied mathematics that go beyond these topics and, most importantly, from the so-called 'formal sciences' (p. 82). It is argued that, by focussing more on formal sciences, such as control theory, data analytics, network theory, information theory, game theory, and theoretical computer science, philosophers of mathematics can gain a much deeper insight into the nature of mathematics (pp. 84–89). This may seem strange, since these sciences, at face value seem to go beyond the usual remit of mathematics. However, their omission from usual taxonomies of applied mathematics is taken to be a mere historical quirk arising from the fact that their applications predated the development of the relevant formalisms. Furthermore, they provide clear examples of formal mathematics that can provide us with 'certainty about the real world' (p. 90). Even if one takes issue with the way that Franklin categorises these 'formal sciences', the idea that philosophers of mathematics should pay more attention to forms of

[1]See [Ladyman and Ross, 2007].

mathematical practice that go beyond their 'traditional diet' deserves heeding.

Having set out a large proportion of the Aristotelian metaphysical picture, Franklin takes a step back, in order to assess where his position stands with respect to other positions in the philosophy of mathematics and to defend it against some standard objections. He provides an interesting response to Frege's famous critique from the *Grundlagen*, which for too long has been mistakenly seen as the final death knell for any philosophy of mathematics that tries to explain things in physical terms (pp. 101–104). He goes on to detail a number of problems that arise from being constrained by the Platonist/nominalist dichotomy, including an overemphasis on the significance of set theory, the infinite, and mathematical indispensability and an underemphasis on the significance of concrete practices such as measurement. Hellman's modal structuralism is taken to be the closest position in the recent literature to the one put forward, in the sense that it is anti-Platonist and replaces talk of abstract entities with talk of possible structures. However, Franklin argues that his own position is superior, since Hellman's modalities are 'ungrounded and in need of a realist theory of universals even to explain what they apply to' (p. 119). The chapter concludes by comparing the approach on offer with a number of naturalist anti-Platonist theories with which it shares some significant properties, ranging from Aristotle, through Newton and Mill to more recent proposals of Maddy, Kessler, Armstrong, Bigelow, and Giaquinto.

Chapter 8 offers a somewhat brief treatment of an Aristotelian account of the infinite. One of the more interesting and controversial aspects of this section arises from questioning the extent to which the infinite is even required for a successful mathematics. Attempts are made to dampen the controversy by suggesting that there are no paradoxes in the notion of infinity and that there is no clear distinction between the notions of potential and actual infinity (pp. 134–137). However, in doing so the author may have raised as many new controversies as he has put down. Given the highly controversial nature of the claims made, much more rigorous defence is required and it would be interesting to see how far the author could go by following through the claim that 'in most of mathematics, infinity is a luxury' (p. 134). As the author beautifully puts it, 'smooth functions, like smooth chocolate, are our preference, but we can cope with the gritty variety if need be' (p. 143). This may be a true reflection of mathematical practice but it calls into question the extent to which Franklin is really offering a realist as opposed to an instrumentalist position.

Having touched on the notion of infinity, Franklin then turns to the issue of geometry (Ch. 9). He argues that it is wrong to take geometry to be inherently concerned with space by pointing to examples of non-spatial 'spaces', such as colour-space. He then turns to the substantivalist *vs.* relationist debate in the philosophy of physics to try to recover the idea that geometry applies to actual space. This section is again quite short, given the scope of the issues addressed and their controversial nature. However, the picture that emerges is convincing and adds to the sense that Franklin has developed a coherent theory that can apply to the whole range of mathematics and its various applications. Furthermore, he provides a new take on the discovery of non-Euclidean geometries as a historically significant episode in the history of mathematics.

Having established the central aspects of his metaphysical position, in Chapters 10–12 Franklin goes on to outline an Aristotelian epistemology of mathematics. Chapter 10 focuses on the role of perception, chapter 11 on the role of imagination, and chapter 12 on the role of the intellect. One of the main benefits of the approach on offer is the possibility of providing a truly naturalist epistemology, which acknowledges the significance of recent empirical research into the nature of mathematical cognition and perception for our understanding of mathematical knowledge. One feels as though much more could be said on the matter, particularly given the recent proliferation of research into mathematical cognition. Certain somewhat controversial issues in the cognitive sciences, such as the differences between classical AI and neural networks, are glossed over and one has a nagging feeling that a more detailed investigation of the cognitive mechanisms invoked might invalidate the theory on offer. At times, despite appeals to empirical evidence, it feels as though the main support for his claims comes from his own introspective assessments, rendering the claims somewhat lacking in naturalistic credentials. One example of this is the author's claim that we can perceive similarities of structure amongst inputs from different sensory modalities (pp. 178–179). Whilst this idea is both fascinating and intuitively compelling, it is highly controversial and requires far more empirical support. However, given the author's expertise as a mathematician rather than a cognitive scientist it seems fair to allow for some minor inaccuracies and deficiencies in terms of detail. The most significant consequence of these sections is the emphasis on the importance of the cognitive sciences for our understanding of mathematical knowledge. To understand mathematical knowledge we must understand the natural processes through which it is acquired. Even if it turned out that a more detailed investigation of the cognitive sciences literature failed to yield support for the Aristotelian approach, a huge amount would have been gained by bringing the significance of this literature to the fore.

Of particular interest is the focus on perceptual access to mathematical content, which arises from our natural ability to perceive directly quantity, symmetry, and isomorphism. This allows Franklin to sidestep Benacerraf's infamous access problem, in a manner similar to the early work of Maddy [1990], yet more closely tied to contemporary empirical research. By paying closer attention to the empirical literature, Franklin is able to go beyond other naturalists, such as Shapiro and Resnik, who merely point to the existence of pattern recognition, and begin to give an account of exactly what pattern recognition consists in. Perception of mathematical properties is taken to be no 'different from the perception of colour' (p. 179) and our access to unperceivable mathematical properties is, in part, taken to be mediated by the same kinds of imaginative process that would allow us to conceive of unperceived colours. Such claims sit well with the view from neuroscience according to which 'number appears as one of the fundamental dimensions according to which our nervous system parses the external world' [Dehaene, 1997, p. 71]. However, in making this comparison Franklin leaves his view open to the kind of anti-realist challenge that is commonplace in the domain of colour, even once contemporary empirical evidence has been taken into account.[2]

Chapter 12 offers a novel account of the role of proofs in mathematics. Unlike others who tend to give formal proof a unique and privileged role in the epistemology of mathematics, Franklin argues that the certainty involved in formal proofs is parasitic on the kind of certainty we associate with direct perception. He argues that when 'mathematical truth is too complex to be visualised and understood at one glance, it may be established conclusively by putting together two glances, or three, or n' (p. 192). Thus proof is no more than a series of perceptually derived certainties. This perspective leads Franklin to provide an interesting critique of traditional positions that put a lot of weight on the significance of proof. For example, formalism is brilliantly chastised for 'mistaking the finger for what is being pointed at' (p. 200). The chapter ends by raising the problem of knowledge of the infinite. The author acknowledges that none of perception, imagination, and proof, as he has described them, are sufficient to provide knowledge of the infinite. He offers an account of how we might come to know about the possibility of infinity through our experience with physical space. However, he admits that different arguments are required to account for so-called higher infinities and yet fails to provide any. Given these difficulties and those addressed in chapter eight, one wonders whether the author might have been better off opting for some form of finitist approach, on which such problems regarding higher infinities would simply fail to apply.

Having provided the beginnings of an account of an Aristotelian epistemology for mathematics, Franklin goes on to address the issue of explanation in mathematics (Ch. 13). He makes the exciting and novel move of trying to unite debates about explanation from the philosophy of science and the philosophy of mathematics, which had hitherto unnecessarily been considered as distinct and unrelated issues. The Aristotelian realist position offers to bridge this gap in both directions by suggesting that accounts of explanation in science and mathematics both have important insights to contribute to each other.

Chapter 14 addresses idealisation, which is one of the seemingly more problematic issues for Aristotelianism. At face value the notion of idealisation seems to provide motivations for Platonism, since idealisations seem to involve entities, such as perfect shapes, that do not exist in reality. Franklin provides four responses to the apparent problem of idealisation. Firstly, he argues that there are many cases where mathematics applies directly to reality without any need for idealisation. Secondly, he argues that many cases of apparent idealisation are best understood as cases of approximation, where simpler structures, such as perfect circles or frictionless planes, are used to approximate the more complex, and no less mathematical, structures that occupy the real world. Thirdly, he argues that cases that seem less like approximations, such as the negative and complex numbers, can still be understood in terms of modelling the world in terms of simple mathematical structures, where the use of these structures is dependent on their mathematical relation to real world structures. Finally, Franklin admits that entities such as zero and the empty set might best be un-

[2] *E.g.*, [Hardin, 1988; Averill, 2005].

derstood as useful fictions but that this need not threaten the viability of the overall realist approach.

The final chapter of the book raises the controversial topic of non-deductive logic in mathematics. The fact that, on the Aristotelian picture, 'mathematics is a scientific study of a world "out there" ' means that it should be seen as '(among other things) an experimental science' (p. 241). Franklin provides a number of examples from the history of mathematics where evidence has been brought to bear on a mathematical problem, in a manner that more closely resembles the empirical sciences than one might initially expect. He then goes on to explain the possibility of probabilistic relations between necessary truths. He finishes by suggesting that the problem of induction arises in mathematical contexts, providing the example of the randomness of the digits π as something that is believed on the basis of inductive rather than deductive means.

The book covers an impressively wide range of different issues in the philosophy of mathematics, providing refreshingly novel takes on a number of different problems. However, there is one issue that is somewhat conspicuous by its absence, namely, the question of mathematical truth. By bringing some of the objects of mathematics down to earth, Franklin is able to avoid the second horn of Benacerraf's dilemma from his landmark paper [1973] 'Mathematical truth'. Our access to mathematical knowledge can be seen as on a par with our access to knowledge of everyday objects, since both ultimately rely on the same kinds of processes. However, in taking this position, Franklin seems to fall foul of the first horn of Benacerraf's dilemma by failing to provide a unified semantics for mathematical claims. Some of the truths of mathematics seem to refer to actual physical manifestations of quantity and structure, whilst others seem to refer to merely possible, uninstantiated mathematical properties. As such, there seems to be no principled way to assess the truth of mathematical claims from the perspective of mathematics alone. There are a number of issues that arise from this, all of which merit further detailed investigation. Firstly, it is not entirely clear what kind of realism Franklin is trying to defend. The absence of a unified semantics seems to threaten the possibility of semantic realism, since the truth of some mathematical claims seems to depend on the existence of mathematical properties whilst others depend on their mere possible existence. Obviously, semantic realism is far from the only option. However, it would be interesting to know more about exactly what form of realism Franklin has in mind to replace it.

A second issue arises out of a possible solution to this problem. Franklin could sidestep the worries associated with Benacerraf's first horn by arguing that, deep down, all mathematical claims are modal claims and, thus, that mathematical truth always depends on the mere possibility of physical instantiation of mathematical properties. As Putnam pointed out [1967], such a move need not be seen as revolutionary, in the sense that mathematical practice would be unaffected. Furthermore, Franklin could help himself to Hellman's [1989] modal semantics for mathematical claims. This would seem like a good move to make, given the acknowledged proximity between Hellman's views and his own. However, the question then arises as to what Franklin gains over the modalist positions of Putnam and Hellman by adopting the further ontological extravagance of commitment to actual physical instantiations of mathematics. In other words, if all mathematical statements are about what is possible then why bother sacrificing ontological innocence and making the further claim that some such possibilities are actual?

An intriguing alternative possibility is that Franklin would be happy to give up on a unified semantics for mathematics altogether. Some mathematical claims could be seen as being committed to the actual existence of certain mathematical structures, whilst others could be committed to the mere possibility of mathematical structures. An intriguing upshot of such a view would be that questions of mathematical ontology could no longer be decided from the perspective of mathematics alone. Insights from the physical sciences regarding the nature of the physical realm, and from the cognitive sciences regarding our access to it, could both be brought to bear on the question of which mathematical structures are actual and which merely possible. One of the benefits of Franklin's position is in opening up room for lively internal debate amongst Aristotelians about where best to draw the line between actuality and possibility. Another tantalising prospect of giving up on universal semantics is the possibility of denying the truth of previously widely accepted mathematical claims. Franklin tentatively suggests the seemingly radical possibility that 'one might accept the reality of $\aleph_0$ but not of any higher infinities' (p. 206). Such a move might be necessary if one thought that physical instantiation of such higher infinities were, in some sense, impossible. Many would find denying the truth of claims about higher infinities too controversial and revolutionary to be palatable. However, it would be extremely interesting to see what an Aristotelian position that wholly embraced such a move would look like. Again, the most important consequence of the possibility of such a move is to highlight the diversity of potential Aristotelian positions available, suggesting that, even once one buys into the Aristotelian framework, there is still room for healthy disagreements and debates.

An Aristotelian Realist Philosophy of Mathematics is a bold attempt to provide comprehensive and consistent position in the philosophy of mathematics that breaks the constraints of recent orthodoxy. By opening up the middle ground between the all-or-nothing dichotomy of Platonism and nominalism, Franklin exposes a whole new terrain of debate. The view expounded is by no means the only Aristotelian position available. One can imagine the outlines of alternative, less realist, Aristotelian positions that disagree about how much of mathematics is actually instantiated or about how much of mathematics can possibly be physically instantiated. One of the strengths of Franklin's work is to reignite neglected debates of this kind. The other main strength lies in rescuing philosophy of mathematics from its solitary isolation by highlighting the need to look to subjects beyond pure mathematics, such as the formal sciences, the cognitive sciences, and physics in order to gain a full understanding of what mathematics is about.

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