SETS AS MEREOLOGICAL TROPES

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Metaphysica 3 (2002): 5-10

Either from concrete examples such as tomatoes on a plate, an egg carton full of eggs and so on, or simply because of the braces notation, we come to have some intuitions about the sorts of things sets might be. (See Maddy 1990.) First we tend to think of a set of particulars as itself a particular thing. Second, even after the distinction between set-theory and mereology has been carefully explained we tend to think of the members of a set as in some sense parts. And third we tend to think that there is something represented by the braces. Now if there were experts who got their intuitions from elsewhere then we could discard these rather crude ideas about egg cartons and so on. But I suspect the intuitions of experts are, just like those of the rest of us, based on notation and simple examples.

Doing full justice to our homely intuitions about sets might well require that we abandon classical mereology and treat the members of a set as quite literally parts of the set. In this paper, however, I explore an alternative, in which sets are identified with mereological tropes. To motivate this account let us first ask what is the difference between {a,b} and a + b , as it might be the set whose members are two volumes of a dictionary and the dictionary itself which is the sum of the two volumes? One difference is that we might think of the braces {,} as somehow standing for something. But more important is that initially at least when we divide up {a,b} a and b remain intact, whereas a + b can be divided up directly into the parts of a and b , say the 1,200 pages.

This suggests the following intuitively appealing account of sets: a set is something considered as having *these* parts (its members) rather than as having *those* parts. Of course considering it a certain way cannot stop it having other parts, but we are to ignore its other parts. This account might be worth developing as a substitute for realism about sets, but a realist theory it is not, for what we "consider" is mind-dependent. So for a realist theory of sets we should seek something real corresponding to something, c, considered as having characteristic F. There seem to be two candidates, the states of affairs *that c is F* or the trope *c's F-ness.* I am suggesting, then, that a set whose members are a, b etc is c-with-a-b-etc-as-parts for some c which has a, b etc as parts. And we have yet to decide if c-with-a-b-etc-as-parts is to be taken as the state of affairs *that c has a, b etc as parts* or as the trope *c's having a, b etc as parts*

Let us consider again {a,b}. In this paper I shall follow Lewis (1991) and take {a,b} to be {a} + {b}. The singleton {a} is the abstract particular c-with-a-as-a-part, {b} the abstract particular d-with-b-as-a-part, and {a,b} is the abstract particular e with-a-and b-as-parts. Whether they are tropes or states of affairs the Lewis equation {a,b} = {a} + {b} holds if and only if c = d = e. There is then only one plausible candidate for c, namely the sum of everything – call it Ω – and we may think of the braces as referring to the sum of everything disjoint from every member of the set.

The hypothesis, then, is that a singleton {a} is either to be taken as the state of affairs *that* Ω *has a as part* or as the trope Ω*'s having a as part.* What is the difference between *that b is F* and *b's F-ness*? One difference, perhaps the only difference, is this: *that b is F* has b as a constituent, whereas *b's F-ness* is itself a part of b. So now let us consider sets of sets, such as {{a}}. Ω would be a constituent of the state of affairs *that* Ω *has a as part.* Hence this state of affairs cannot itself be proper part of Ω. For any part is a constituent and the only way in which x and y can be constituents of each other is if x = y. Therefore there could be no such entity as {{a}}. However this problem does not arise in the case of tropes. For tropes of the form Ω*'s being F* are parts of Ω. We reach the conclusion then that this intuitively motivated account of sets requires sets not to be identified with states of affairs but rather with mereological tropes which are sums of ones of the form **Ω***'s having z as a part*, where z ranges over all members of the set in question.

If we adjoin the null element ø, classical mereology becomes a complete Boolean algebra, so whether or not we are realists about ø it is convenient to adjoin it. In that case ø will also perform the role of the null set Ø and {Ø} will be **Ω**'s having ø as a part.

Do we have a set {**Ω**}? That would be **Ω**'s having **Ω** as a part. So do we mean "part" or "proper part"? We could stipulate either way, but there is something peculiar about allowing tropes of the form: x's having x as a part. So take "part" to mean "proper part."

Problems

The first and most obvious problem with treating sets as mereological tropes is that there seem to be necessitation relations between tropes, so that **Ω***'s having b as a part* necessitates **Ω***'s having c as a part* if c is part of b. For nothing can have b as a part without having c as a part too. But sets with b as a member do not always have c as a member. Likewise the sum of the tropes **Ω***'s having b as a part* and **Ω***'s having d as a part* necessitate, the trope **Ω***'s having (b + d) as a part.* But sets with b and d as members do not always have b + d as a member too. The solution to this problem is to grant that one trope x can necessitate another trope y without y being part of x. If b is part of c then the existence of **Ω***'s having b as a part* entails the existence of the trope **Ω***'s having c as a part.* But all this shows is that the existence of {b} implies the existence of {c}, as it should.

Another problem faced by a trope theory of sets is that we might have to use sets as a tool in trope theory. We should check that this results in neither confusion nor absurdity. Thus if we identify a universal with a set of tropes we will find that the universal itself becomes a rather special trope, and instantiation has a mereological interpretation. Consider the universal *being a sphere*. This would be identified with a set of resembling tropes, *a's being a sphere, b's being a sphere* etc. That in turn is to be identified with the sum of: Ω*'s having as part a's being a sphere,* Ω*'s having as a part b's being a sphere* etc. If Ω were replaced in this analysis by something smaller, such as the sum of all spheres, then this would be a counter-intuitive identification. It is, however, perfectly appropriate that Ω – everything – should be the "locus" of the trope which we identify with a universal.. So there seems to be no problem here either.

The third problem is both a threat and an opportunity. An ontology of sets should resist paradox without ad hoc manoeuvres. If it does then this supports the account given. And in this case we are able to resist paradox by appealing to philosophical intuitions which pre-date set theory. For mereological tropes exist *in virtue of* other tropes, that is they are both entailed and explained by other tropes. For instance {b}, that is the trope of Ω*'s having b as a part,* exists in virtue of the tropes of which b is the sum. To avoid paradox we may state a general Well Founding Principle, namely that the existence-in-virtue-of relation must be well-founded. Assuming the Axiom of Choice this amounts to denying an infinite regress of x_1 existing in virtue of, among other things, x_2 which exists in virtue of, among other things, x_3 etc. Such a rejection of an infinite explanatory regress is independently plausible and predates set theory. It is for instance a premise used in cosmological arguments for the existence of God. Sets considered as mereological tropes are subject to this requirement, which prevents there being paradoxical sets.

Comparisons

We may conclude by comparing the identification of sets as mereological tropes with other recent accounts of sets. If we aim to follow Lewis (1991) in treating subsets as parts of sets and if we aim to treat sets as particulars then there are two other accounts available. First there is Lewis's own primitivism about the singleton operator. If we can avoid such primitivism we should do so and hence if we are otherwise inclined towards a trope theory we should prefer the mereological tropes thesis.

The other recent account is Armstrong's(1991) . On it sets are states of affairs, whose atomic parts are the singletons, where {b} is the state of affairs of b having unithood, where *unithood* is the property of having some unit-determining propert,y and where a *unit-determining* property is one such that 'a thing falling under the property is just *one* thing of that sort. . . .' (Armstrong, 1991, p. 197.) Now any non-class can be a member of a set, but is any non-class a unit? It would seem possible for there to be an entity with parts exactly resembling the whole, and hence parts of parts exactly resembling the whole, and so on. In that case what property could any part have such that anything with that property was just *on*e thing with that property? For example might there not be a sort of homogenous stuff which was made up of what we take to be point particles but where every such "point particle" is in fact itself a whole "universe" full of homogenous stuff exactly resembling the stuff we started off with. In that case it would in turn be made of "point particles" which in fact are whole "universes" full of the same sort of stuff and so on. Hence any property of one of these "point particles" is also instantiated by all the "point particles" which are its parts, preventing there being any unit-determining properties. Less fancifully, the failure of the Identity of Indiscernibles makes it likely that there can be infinite populations of indiscernible entities, say an infinity of electrons, in which case there will be no property of the sum of all those electrons not instantiated by any part which is also the sum of an infinity of electrons.

I am not suggesting that I have considered all the intuitively acceptable identifications for sets. Indeed I have already alluded to an appeal to non-classical mereology. But what I have shown is that trope-theorists have an account which is preferable to the best known rivals.

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