Expander Graphs, Strong Ergodicity, and Superstrong Approximation

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Abstract. After introducing expander graphs and briefly discussing classical results and constructions, I will talk about recent developments pertaining to establishing expansion property for congruence quotients of thin groups –discrete subgroups of semi-simple groups which are Zariski dense but of infinite index. This expansion property is intimately related to strong ergodicity of the associated group action and can be viewed as a far-reaching generalization of the strong approximation theorem.

Recent Results in Pattern-avoiding Permutations

Tony Guttmann
The University of Melbourne

Abstract. In the late 1960s Don Knuth asked which permutations could be sorted by various data structures. For a simple stack, he proved that permutations avoiding the pattern 312 can be so sorted, and further, that the number of such permutations of length n is given by C_n, the n^{th} Catalan number. Knuth went on to pose the same question about other data structures that are widely used in sorting operations, namely deques, and two stacks, both in parallel and series. All these problems remain unsolved to this day, though there have been important developments. However while it is known that the number of permutations sortable by these structures grows exponentially, with a structure-dependent growth constant, the value of this constant is unknown in each case. We will discuss our efforts to estimate the constant in a number of cases, and also discuss the sub-leading asymptotic behaviour.

Knuth's work and subsequent developments saw the area of pattern-avoiding permutation develop into an interesting area of combinatorics in its own right. The fundamental question asked is "how many permutations of length n avoid a given pattern?" Here the *pattern* is a sub-permutation. The answer to this question is known for the $3! = 6$ patterns of length three. In every case the answer is given by the n^{th} Catalan number. For the $4! = 24$ permutations of length four, it is known that these fall into 3 distinct classes. Two of these have been completely solved, and it is known that the growth constants in the two solved cases are 8 and 9 exactly. For the third class, those avoiding the pattern 1324, very little is exactly known. By extensive computer enumeration, and careful numerical work, we show that the

growth constant is 11.60 ± 0.01 and obtain a good estimate of the asymptotic behaviour, which is quite different to that for the two solved cases.

Finally, we comment on permutations avoiding the vincular pattern $abcd{-}e$. Several cases of such patterns remain unsolved. From extensive enumerations we are able to make conjectures for the exact value of the growth constants for several of the unsolved cases. These turn out to be transcendental numbers. We then prove that the corresponding generating functions cannot be D-finite, contrary to earlier conjectures that all such patterns are D-finite.

This is joint work with Andrew Conway.

Cayley Graphs of Finite Groups

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Abstract. A Cayley graph on a finite group can be viewed as a dynamical system on the group, with a given specified generating set. Such graphs have been used as models for complexity analyses of randomized group computations, and contribute to the problem of analyzing the complexity of graph isomorphism. The diameters of Cayley graphs give a measure of the ‘mixing time needed for randomly traversing or sampling from the group, and a long-standing conjecture of Babai specifies how large the diameter may be in terms of the group order (especially for simple groups). Results on growth in groups, independently by BreuillardGreenTao and PyberSzabo yield proof of Babais conjecture for bounded rank Lie type groups, and the best result for finite alternating and symmetric groups come very recently from HelfgottSeress. I will discuss these results and how they yield the best current results towards proof of other conjectures on non-regular group actions. If there is time Ill also discuss an approach to describing the structure of edge-transitive Cayley graphs the role played by simple groups, and how much additional symmetry is possible.

Sandpiles and Harmonic Systems

Klaus Schmidt
University of Vienna

Abstract. The d-dimensional abelian sandpile model is a lattice model introduced in 1987 by Bak, Tang and Wiesenfeld as an example of what they called ‘self-organized criticality’. Although this deceptively simple model has been studied quite intensively both in the physics and mathematics literature, some very basic questions about it are still open, like its properties under two different kinds of dynamics: ‘addition’ (of grains of sand), and the shift-action.

By extending an algebraic construction originally introduced by A. Vershik for Markov partitions of hyperbolic automorphisms of the 2-torus one can show that the sandpile model is closely related to a certain $\mathbb{Z}^d$-action by automorphisms of a compact abelian group, the 'harmonic model'.

The purpose of this lecture (which is based on joint work with Evgeny Verbitskiy) is a discussion of this construction and of the conclusions that can be drawn from the connection between these systems.

The Arithmetic Complexity of Orbits

Joseph H. Silverman
Brown University

Abstract. A naive measure of the arithmetic complexity of a number, or more generally of a point $P = (x_1, ..., x_N)$ with algebraic coordinates, is the number of bits required to store P on a computer. This quantity is called the *height of* P and is denoted $h(P)$. Let X be a quasi-projective variety, let P be a point of X, and let $f : X \to X$ be a map given by rational functions, with everything defined over a number field. Then the growth rate of $h\big(f^n(P)\big)$ provides an interesting measure of the arithmetic complexity of the forward orbit of P under iteration of f. In favorable situations, the *canonical height* limit $\hat{h}_f(P) = \lim_{n\to\infty} h\big(f^n(P)\big)/\delta_f^n$ exists and has good properties, where the *dynamical degree* $\delta_f = \lim_{n\to\infty}(\deg f^n)^{1/n}$ measures the geometric complexity of the iterates f^n. Even if $\hat{h}_f(P)$ does not exist, we may consider a coarser growth invariant, the *arithmetic degree* $\alpha_f(P) = \lim_{n\to\infty} h\big(f^n(P)\big)^{1/n}$. **Conjecture 1**: The limit $\alpha_f(P)$ exists, is an algebraic integer, and takes on only finitely many values as P varies. **Conjecture 2**: If the forward orbit of P is Zariski dense in X, then $\alpha_f(P) = \delta_f$. We will discuss these definitions and conjectures and the recent proof of Conjecture 1 when X is projective and f is a morphism and of Conjecture 2 when X is a torus or an abelian variety and f is an isogeny. (Joint work with Shu Kawaguchi).

Renormalization in Two-dimensional Piecewise Isometries

Franco Vivaldi
Queen Mary, University of London

Abstract. Piecewise-isometries are zero-entropy dynamical systems with very complex behaviour. In one-dimension they are interval-exchange transformations (IET), which are connected to diophantine arithmetic by the Boshernitzan and Carrol theorem: in any IET defined over a quadratic number field, the process of induction results,

after scaling, in an eventually periodic sequence of IETs. This can be viewed as a generalization of Lagrange's theorem on the eventual periodicity of the continued fractions of quadratic irrationals.

In two dimensions we have polygon-exchange transformations (PET). Until recently, their study has been limited to specific systems defined over quadratic fields, which, invariably, have been found to exhibit eventual periodicity. I'll describe some recent results in parametrized families of PETs, where the (fixed) rotational component identifies a quadratic field. We show that a suitable induction eventually generates a scaled version of the original map, re-parametrized by a Lüroth-type function —a piecewise affine version of Gauss' map. The parameter values corresponding to exact scaling are found to be precisely the elements of the underlying quadratic field. The proofs required computer-assistance. (Joint work with J. H. Lowenstein).

Rigid and flexible invariants for group automorphisms

Thomas Ward
Durham University

Abstract. The structure of the space of all pairs (G, T), where G is a compact metric group and $T : G \to G$ a continuous ergodic automorphism can be studied via dynamical invariants. We will discuss some of the issues that come up in trying to understand which invariants can vary flexibly and which exhibit rigidity in moving through this space.

In particular, we will describe how many of these questions reduce to issues like Lehmer's problem or constructions of sets of primes with prescribed properties.

Random graphs, random regular graphs and combs

Nick Wormald
Monash University

Abstract. When a network grows randomly, the point at which it achieves a given property can often be pinpointed in advance with high probability. In their early work on random graphs in the late 50's, Erdős and Rényi considered the threshold of appearance of a giant component, and of various other subgraphs.

The models of random graphs introduced at that time have received much attention since then, and have found many applications, particularly in computer science. Many interesting results can be stated in terms of the random graph process. In this, the random graph grows in time by the addition of random edges, making the graph ever denser as time goes on. The threshold of appearance of a given subgraph H can be defined as the time at which the random graph process contains a copy of H with probability at least 1/2.

Kahn and Kalai made a simple-sounding but deep conjecture relating this threshold to another one: the threshold of expectation for a subgraph H is the time at which the expected number of copies of H in the random graph process exceeds 1. One very special case of this conjecture gained some notoriety as the comb conjecture, made about 15 years ago by Kahn.

Random regular graphs are a different but commonly used model of random graphs with low density. This model enters somewhat surprisingly into a solution of the comb conjecture, recently obtained jointly with Jeff Kahn and Eyal Lubetzky. In this talk I will give an exposition of results on random graphs and random regular graphs, with the proof of the comb conjecture as a focus.