Solutions Version 6b

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1.
$$\mathbf{x} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -7 \\ 0 \\ 1 \end{pmatrix}$$

Parellel vectors include $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -7 \\ 0 \\ 1 \end{pmatrix}$

2. We wish to see whether $\begin{pmatrix} -4 \\ -3 \\ 2 \end{pmatrix}$ can be written as a linear combination of $\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 6 \\ -4 \end{pmatrix}$. Hence,

$$\left(\begin{array}{rr|r} 1 & -3 & -4 \\ -3 & 6 & -3 \\ 2 & -4 & 2 \end{array}\right) \to \left(\begin{array}{rr|r} 1 & -3 & -4 \\ 0 & -3 & -15 \\ 0 & 0 & 0 \end{array}\right)$$

Gaussian elimination (steps not shown) shows there is a solution. The line is parallel to the plane.

3.
$$\left(\begin{array}{ccc|c} 1 & 3 & a & 2 \\ 1 & 4+a & 1 & 0 \\ a & -3 & -5 & 2 \end{array}\right) R_2 := R_2 - R_1 \; R_3 := R_3 - aR_1$$

$$\left(\begin{array}{ccc|c} 1 & 3 & a & 2 \\ 0 & 1+a & 1-a & -2 \\ 0 & -3-3a & -5-a^2 & 2-2a \end{array}\right) R_3 := R_3 + 3R_2$$

$$\left(\begin{array}{ccc|c} 1 & 3 & a & 2 \\ 0 & 1+a & 1-a & -2 \\ 0 & 0 & -(a+1)(a+2) & -4-2a \end{array}\right)$$

By inspection, if $a = -1$ we have no solution. If $a = -2$, we have infinite solutions. Otherwise, we have a unique solution.