Solutions Version 6a

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1.
$$\mathbf{x} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + \lambda \left[\begin{pmatrix} 3 \\ -6 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} \right] = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix}$$

For the Cartesian form,. $x_1 = 1 + 2\lambda$, $x_2 = -2 - 4\lambda$, $x_3 = 4$, Removing λ,

$$x_1 - 1 = \frac{-x_2 - 2}{2}, \ x_3 = 4.$$

2. Setting the lines equal to each other,

$$\begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix},$$

$$\lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix},$$

$$\left(\begin{array}{cc|c} 2 & -2 & 2 \\ 1 & -2 & -1 \\ 4 & -5 & 2 \end{array}\right)$$

By Gaussian elimination (not shown), we find there is a (unique) solution. Hence, the lines intersect.

3. Setting the plane and the line equal to each other,

$$\begin{pmatrix} 1 \\ -5 \\ 7 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} + \beta \begin{pmatrix} -2 \\ -7 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ -6 \end{pmatrix}.$$

Rearranging as an augmented matrix, (α, β, λ),

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 4 & -7 & -3 & 6 \\ -2 & 3 & 6 & -9 \end{array}\right) R_2 := R_2 - 4R_1, \ R_3 := R_3 + 2R_1$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & -7 & 2 \\ 0 & -1 & 8 & -7 \end{array}\right) R_3 := R_3 + R_2$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & -7 & 2 \\ 0 & 0 & 1 & -5 \end{array}\right)$$

Hence, $\lambda = -5$, (and $\beta = -33$, $\alpha = -60$), so $\mathbf{x} = (7, \ -14, \ 28)^T$.