Solutions Version 2a

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1.
$$x_1 = -2 + 4\lambda, \quad x_2 = -5 + 6\lambda, \quad x_3 = 7 - 3\lambda,$$

hence, (λ=),

$$\frac{x_1 + 2}{4} = \frac{x_2 + 5}{6} = \frac{-x_3 + 7}{3}.$$

Setting the point on the line gives

$$\begin{pmatrix} -2 \\ 5 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix} = \begin{pmatrix} 50 \\ 73 \\ -34 \end{pmatrix}$$

The first row gives $\lambda = 13$, and the last, $\lambda = 12\,2/3$, a contradiction. The point does not lie on the plane.

2. Setting the point in the plane gives

$$\begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$$

As an augmented matrix we then have

$$\left(\begin{array}{cc|c} 1 & 3 & -2 \\ -3 & 2 & 0 \\ -2 & -4 & -2 \end{array}\right)$$

Which we find, after Gaussian elimination (not shown), not to have a solution.

3.
$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 4 & 1 \\ 1 & 3 & -3 & 3 & 3 \\ 6 & 15 & -17 & 19 & 15 \end{array}\right) R_2 := R_2 - R_1,\ R_3 := R_3 - 6R_1$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 4 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 3 & 1 & -5 & 9 \end{array}\right) R_3 := R_3 - 3R_2$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 4 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 & 3 \end{array}\right)$$

let $x_4 = \lambda$, then $x_3 = 3 + 2\lambda$, $x_2 = 2 + \lambda$, $x_1 = 6$.