MATH1131 TEST 2 VERSION 3b

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1. To find the parametric form of the plane in $\mathbb{R}^3$ defined by $2x_1 - 4x_2 + x_3 = 7,$ we set

$$\begin{aligned} \lambda &= x_2, \\ \mu &= x_3. \end{aligned}$$

Then the equation of the plane tells us that

$$x_1 = 7/2 + 2\lambda + \mu/2;$$

Thus the parametric form is given by

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7/2 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix}, \text{ for } \lambda, \mu \in \mathbb{R}.$$

Thus two non-zero non-parallel vectors which are parallel to the plane are $(2 \ \ 1 \ \ 0)^{\perp}$ and $(\frac{1}{2} \ \ 0 \ \ 1)^{\perp}$.

2. The lines intersect if and only if there exist $\lambda, \mu \in \mathbb{R}$ such that

$$\begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}, \ i.e.$$

$$\lambda \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}.$$

We can row reduce the corresponding augmented matrix to

$$\left(\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right)$$

which has a unique solution, implying the lines do indeed intersect.

3. Assuming that $p(x) = ax^2 + bx + c$, the data $p(1) = 4, p(2) = -3$ and $p(-2) = 1$ tell us

$$\begin{aligned} a + b + c &= 4 \\ 4a + 2b + c &= -3 \\ 4a - 2b + c &= 1 \end{aligned}$$

Row reducing the corresponding augmented matrix yields

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 7 \end{array}\right),$$

implying $a = -2, b = -1, c = 7$. Thus $p(x) = -2x^2 - x + 7$.