MATH1131 TEST 2 VERSION 3a

by hugo bowne-anderson

1. To find the parametric form of the line

$$\frac{x_1+5}{-1} = \frac{x_2}{-2} = \frac{x_3+7}{-5},$$

we let all terms in the cartesian equation equal λ, that is, we set

$$\frac{x_1+5}{-1} = \frac{x_2}{-2} = \frac{x_3+7}{-5} = \lambda.$$

Then we see that

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ -5 \end{pmatrix}, \quad \lambda \in \mathbb{R}.$$

Then $\boldsymbol{a} = \begin{pmatrix} -5 \\ 0 \\ 7 \end{pmatrix}$ is a point on the line and $\boldsymbol{b} = \begin{pmatrix} -1 \\ -2 \\ -5 \end{pmatrix}$ is a vector parallel to the line.

2. $\begin{pmatrix} -5 \\ 8 \\ -1 \\ 6 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ -1 \\ 4 \end{pmatrix}$, and $\begin{pmatrix} -1 \\ 2 \\ 1 \\ -2 \end{pmatrix}$ if and only there exist $\lambda_1, \lambda_2, \lambda_3$ such that

$$\lambda_1 \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1 \\ 0 \\ -1 \\ 4 \end{pmatrix} + \lambda_3 \begin{pmatrix} -1 \\ 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -5 \\ 8 \\ -1 \\ 6 \end{pmatrix}.$$

The augmented matrix form of this system of linear equations

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 1 & 0 & 2 & 8 \\ 0 & -1 & 1 & -1 \\ -2 & 4 & -2 & 6 \end{array}\right)$$

row reduces to

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & -5 \\ 0 & 1 & 3 & 13 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

which has a unique solution implying yes!: $\begin{pmatrix} -5 \\ 8 \\ -1 \\ 6 \end{pmatrix}$ is *a* linear combination of $\begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ -1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 1 \\ -2 \end{pmatrix}$.

3. The line and the plane intersect for values of λ, α, β such that

$$\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -8 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix}.$$

Reducing the corresponding augmented matrix (with β, α and λ in the 1st, 2nd and 3rd columns respectively) yields

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -14 \\ 0 & 1 & 0 & -15 \\ 0 & 0 & 1 & 3 \end{array}\right).$$

Therefore intersection occurs when $\lambda = 3$; substituting this into the equation of the line yields the intersection point

$$\boldsymbol{x} = \begin{pmatrix} 0 \\ -23 \\ 12 \end{pmatrix}.$$