

MATH1131 TEST 2 VERSION 2b

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1. i) The vector

$$\begin{aligned}\overline{AB} &= \overline{OB} - \overline{OA} \\ &= \begin{pmatrix} 8 \\ 9 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ 12 \\ -6 \end{pmatrix}\end{aligned}$$

ii) First notice that the point D lies on the line through A and B , which is given by

$$\mathbf{x} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 12 \\ -6 \end{pmatrix};$$

Since $\overline{AD} = 2\overline{DB}$, D lies $2/3$ of the way from A to B and is thus given by the point on the line when $\lambda = 2/3$:

$$\begin{aligned}\mathbf{d} &= \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 8 \\ -4 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ 5 \\ -3 \end{pmatrix}.\end{aligned}$$

2. The plane through the points $\mathbf{a} = \begin{pmatrix} 7 \\ 0 \\ -4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -2 \\ 0 \\ -6 \end{pmatrix}$ is given by the parametric equation

$$\begin{aligned}\mathbf{x} &= \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) \\ &= \begin{pmatrix} 7 \\ 0 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} -9 \\ 0 \\ -2 \end{pmatrix};\end{aligned}$$

thus $\begin{pmatrix} -5 \\ 1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -9 \\ 0 \\ -2 \end{pmatrix}$ are two non-parallel, non-zero vectors parallel to the plane.

3. Row reducing the augmented matrix gives us

$$\left(\begin{array}{ccc|c} 1 & 2 & a & 3 \\ 0 & -1-a & a & -2 \\ 0 & 0 & -(a-3)(a-1) & 2(1-a) \end{array} \right)$$

There are infinitely many solutions when $a = 1$, no solutions when $a = 3$ or $a = -1$ and a unique solution otherwise.