The paper proposes a new rigorous result in the estimation of the metric entropy of a smooth $C^{1+\gamma}$ transitive hyperbolic dynamical system $T: M \to M$, on a compact Riemannian manifold $M$.

It is known that the statistical properties of such a system are described by a “natural” measure, the Sinaï-Bowen-Ruelle (SBR) probability measure $\mu$. The author obtains estimates of (i) the SBR measure, (ii) the $\mu$-Lyapunov exponents of $T$, (iii) the rate of decay of correlations with respect to $C^\gamma$ test functions, and (iv) the pressure (for repellers).

These estimates are proved when $\dim M = 2$ except for expanding systems where they are established in all dimensions. The method relies on the approximation of smooth hyperbolic dynamical systems by Markov chains. Let $(\mathcal{P}_n)_{n \in \mathbb{N}}$ be a sequence of Markov partitions such that $\lim_n \text{diam} \mathcal{P}_n = 0$. Define the matrix

$$(Q_n)_{i,j} = \frac{m(A_{n,i} \cap T^{-1}A'_{n,j})}{m(A'_{n,j})},$$

where $m$ is the volume and $A_{n,k}$ describes the generic atom of $\mathcal{P}_n$. Then $Q_n$ has a largest eigenvalue $\rho_n$ and a strictly positive associated eigenvector $v_n$. Then define the stochastic matrix $(P_n)_{i,j} = (Q_n)_{i,j}v_{n,j}/P_n v_{n,i}$: it has a unique fixed eigenvector $p_n$. Then $\mu_n(\cdot) = \sum_{i=1}^{k_n} p_{n,i}m(A_{n,i} \cap \cdot)/m(A_{n,i})$ converges towards $\mu$, $\lambda_n = -\sum_{i,j=1}^{k_n} P_n(i,P_n)_{ij} \log(Q_n)_{ij}$ estimates the sum of the positive exponents and the $\mu$-entropy of the system is approximated by $h_n = \log \rho_n + \lambda_n$.

These results expand the previous one obtained by the same author; examples illustrate the method. The interest of the method—in contrast to standard techniques using a single long orbit of the system—consists in using information from all regions of the phase space. Unfortunately, in dimensions larger than 2 the numerical construction of Markov partitions may be time-consuming
Reviewed by Bernard Schmitt

[References]

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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