Course Outline

Lecturer/Tutor: Daniel Chan

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Consultation Hours: TBA (see webpage).

Most of the information you need to know about the course can be gotten from the webpage above.

Lectures/Tutorials: There will be 3 hours of classes each week. There will be a tutorial every other week. All other classes will be lectures.

What you need to know to do this course

Ideal preparation for this course is the MATH3711: Algebra course. You need to be familiar with basic concepts in modern algebra and in particular, know a little about groups and rings. I hope the following notions are straightforward for you: abelian groups, quotient groups, isomorphism theorems, group actions, principal ideal domains, field of fractions, characteristic of a field.

About this course

This course is a continuation of MATH3711 and, together with MATH5725: Galois Theory, rounds out the basic curriculum in undergraduate modern algebra. The material in this course is indispensable for anyone interested in number theory or algebra and is considered standard material for any pure mathematician.

Modules generalise the notion of vector spaces to the context where the scalars come from an arbitrary ring. They are ubiquitous in mathematics. A key source of examples, and one we will study in depth is group representations, where a group acts linearly on a vector space. Group representations are a powerful tool, whereby scientists can use symmetry to understand and solve problems in otherwise complicated situations that arise in mathematics and physics.

Assessment

The grade for this course will be determined from 2 short assignments (worth 15% each), and a final exam (worth 70%). The assignments are meant to be relatively straightforward, once you have understood the material. The hard part is of course
understanding the material. It is expected that most of you will be getting close to full marks in the assignments. That way, a “pass” in the final exam should get you close to a credit. If you are having trouble with the assignments, you should talk to other students or to me about it. The most important thing is that you learn the material. The final exam will include questions of a more challenging nature and should distinguish the best students in the class.

**Studying for this course**

This course will be taught in a similar fashion to MATH3711. It is fairly demanding conceptually, but hopefully, you will be getting used to this. The concepts will take a while to digest so don’t expect to understand everything in lectures. Try to get as much as possible out of them, and go over the material regularly after class. I suspect that filling in these gaps in understanding will take up a significant amount of your study for this course. If you are understanding very little of the lectures, then that’s probably an indication that you haven’t properly understood material in earlier lectures. I also strongly suggest you supplement your learning by browsing the references below.

There are few tutorials in this course, so you will be expected to do most of the exercises in your own time. This is to help prepare you for learning later in life, whether in academia or industry, where tutorials are rarely given.

**Student learning outcomes**

Mathematically, I hope you will consolidate your understanding of the fundamentals of modern algebra and gain an appreciation of how deep abstract mathematical concepts can be used to solve concrete problems.

From a skills perspective, I hope you will develop your problem solving and analytical skills. The course should also help you improve your conceptual thinking. You should understand by now, that modern mathematics is communicated in a very different fashion to other disciplines and to everyday speech. Though it can be terse, it has great precision. I hope in this course, you will gain a greater appreciation of the modes of mathematical communication.

**Tentative Syllabus**

- Modules basics: quotient modules, homomorphisms, direct sums, free modules, isomorphism theorems, universal properties
- Modules over a PID: structure theorem and application to Jordan canonical forms
- Chain conditions: noetherian and artinian rings and modules, Hilbert’s basis theorem, Jordan-Hölder theorem
• Wedderburn-Artin theory: semisimple rings, semisimple modules, Schur’s lemma, Wedderburn-Artin theorem

• Some categorical tools: exact sequences, split morphisms, tensor products, adjunction

• Group representations: Maschke’s theorem, characters, orthogonality relations, induced representations, Frobenius reciprocity

References

The lectures will cover all the material that you need to know, but nevertheless, you will probably find it handy to supplement your studies by looking at texts such as those below. There are lots of texts on the subject. They vary a lot so you should scout around for what’s suitable for you.


Continual Course Improvement

The School of Mathematics evaluates each course each time it is run. Feedback on the course is gathered using, among other means, UNSW’s Course and Teaching Evaluation and Improvement Process. Student feedback is taken seriously, and continual improvements are made to the course based in part on such feedback.
In response to student feedback, an extra problem set 0 has been added to review material from MATH3711.

School of Mathematics and Statistics Student Policies

School of Mathematics and Statistics policy regarding tests, assignments additional assessment etc can be found at
http://www.maths.unsw.edu.au/currentstudents/assessment-policies
The UNSW Plagiarism Policy is also there.

Daniel Chan