Revision

**Q1** Diagonalise

\[ A = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]

**Q2** a) Suppose \( A \in M_{nn}(\mathbb{R}) \) has \( n \) distinct e-values. Show that the e-spaces are 1-dimensional.

b) Suppose that \( A \in M_{33}(\mathbb{R}) \) has e-values 0, 1, 2. Find rank \( A \), null \( A \).

**Q3** Consider the dts \( x(k+1) = Ax(k) \) where

\[ A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1.3 & 2.1 & 1.6 \end{pmatrix}. \]

Is the system stable?

**Q4** Define the function \( T : \mathbb{P}_1 \longrightarrow \mathbb{P}_1 \) by

\[ (Tp)(x) = p(x) + p(x + 1). \]

Show that \( T \) is linear but does not have an e-basis.
Q5 Let $S : U \longrightarrow V, T : V \longrightarrow W$ be linear maps.

a) Show that $\ker(T \circ S) \supseteq \ker S$.

b) Show that if $T : V \longrightarrow V$ is linear and $V$ is finite dimensional, then $\ker T^n = \ker T^{n+1}$ for all $n$ large enough.

Q6 Show by induction on dimension that any subspace $V$ of $\mathbb{R}^n$ has an o/n basis. Hint: If $v \in V$ then show that

$$v^\perp := \{w \in V | v \cdot w = 0\}$$

is a subspace of $V$.

Q7 Let $S = \{v_1, v_2, v_3\}$ be a linearly dependent set of vectors whose span is the vector space $V$. Let

$$w_1 = v_1 + v_2, w_2 = v_1 + v_3, w_3 = v_1 + v_2 + v_3.$$ 

a) Is $\{w_1, w_2, w_3\}$ linearly independent?

b) Does $\{w_1, w_2, w_3\}$ span $V$. 