Lecture 5: Complex Polynomials

Aim Lecture Factorise real
by factorising

Remainder & Factor thm

Defn A complex polynomial of degree $n$ is a fn $p$:

$p(z) = a_0 +$

where $a_0$,

If the coeff $a_0, \ldots, a_n$ are

Real poly will also refer to the real-valued

Remainder Thm Let $p(z)$ be a poly &

$\alpha \in \quad$ The remainder
Proof If quotient is \( q(z) \) &

An immediate corollary is

**Factor Thm** Let \( p(z) \) be a poly & \( \alpha \in \)

Then \( z - \alpha \) is a factor of \( p(z) \) iff

**Factorising over \( \mathbb{C} \)**

**Thm** Let \( p(z) = a_n z^n + \ldots + a_1 z + a_0 \) be

a complex poly of degree \( n > 0 \). We can

\[ (*) \quad p(z) = a_n( \]

where \( \alpha_1, \]
Furthermore, the factorisation in (*) is unique up to permuting factors.

**Defn** The number of times a root $\alpha_i$ occurs in the factorisation is called the multiplicity of the root.

**e.g.** 1 $z^4 + 2z^2 +$

So $i, -i$ are

**e.g.** 2 Factorise $4 - z^6$ over $\mathbb{A}$ Use thm. Find solns to

$|z|^6 =$

$6 \text{ Arg } z =$

$\implies \text{ Arg } z =$
So roots are \( z = \)

\[ 4 - z^6 = \]

**Factorising over \( \mathbb{R} \)**

Use

**Propn** a) Let \( p(z) = \sum a_j z^j \) be a real poly

& \( z = \alpha \)

b) \((z - \alpha)(z - \bar{\alpha})\)

which is

Proof 1): If \( 0 = p(\alpha) = \)

then \( 0 = \sum \)
e.g. 2 cont’d Factorise $4 - z^6$ over

A Collect factors corresp

$$(z - \sqrt[3]{2}e^{\pi i/3})(z - \sqrt[3]{2}e^{-\pi i/3}) =$$

Sim $(z - \sqrt[3]{2}e^{2\pi i/3})(z - \sqrt[3]{2}e^{-2\pi i/3}) =$

$4 - z^6 =$

Application to polynomial interpolation

Corollary a) If poly $p(z), q(z)$ have de-
grees \leq n & agree on

b) Any 2 poly which agree on an infinite set

Proof: Clear a) \implies b). Note \( g(z) := p(z) - \)

It also has more than

\textbf{e.g. 2} Given 3 distinct points \((x_1, y_1), (x_2, y_2)\) & \((x_3, y_3)\), there is at most 1 parabola of the form \( y = p(x) \) going through those points.

Why? If \( y = q(x) \) also went through those points then
Symmetric polynomials in the roots

**Defn** A poly $p(x_1, \ldots, x_n)$ in var $x_1, \ldots, x_n$ is symmetric if it remains

**e.g. 4** In 3 var,

$x_1 + x_2 + x_3$ is

$x_1x_2 + x_2x_3$

**e.g. 5** Suppose $z^2 + bz + c$ has roots

$z^2 + bz + c = (z-$

$\implies$ sum of

**Prop** Let $\alpha_1, \ldots, \alpha_n$ be

$p(z) = a_0 + a_1z+$
Then $\frac{a_{n-j}}{a_n} =$

Proof: Just expand

**N.B.** The $a_i$'s are

**Thm** Any symmetric poly in the roots of $p(z)$ is a

No proof.

**e.g.** 6 If $z^3 + 2z^2 + 3z + 4$ has roots $\alpha, \beta$

$\alpha^2+$