Lecture 4: Trig identities. Complex loci.

**Aim Lecture** See how complex numbers can be used to construct sets of points on the plane.

Plot some

**Basic Formulae**

**Facts**

1. \( e^{i\theta} = \)
2. \( \sin \theta = \)
3. \( \cos \theta = \)

**Proof** From picture

2. (3. is similar)
Trigonometric polynomials

Defn A trigonometric polynomial is a

\[ T(\theta) = a_0 + \sum_{n=1}^{N} \theta \]

Rem: 1. If \( N = \infty \), we call it a
2. \( T(\theta) \) is periodic

You can convert any poly in

e.g. 1 Write \( \sin^4 \theta \) as a

A Use fact 2 and

Binomial Thm

\((a + b)^n =\)
\[
\sin^4 \theta = \\
= \frac{1}{16} \left( (\binom{4}{0}) e^{i4\theta} - (\binom{4}{1}) e^{i3\theta} e^{-i\theta} + \\
(\binom{4}{0}) e^{i4\theta} + e^{-i4\theta} \right) \\
= \frac{1}{8}
\]

N.B. $\sin^4 \theta$ is an even

**Uses** $\int \sin^4 \theta$

**Rem** Fourier theory (taught in 2nd yr) shows that any nice fn of period $2\pi$ can be expanded using

**e.g. 2** Write $\sin 3\theta, \cos 3\theta$

**A** De Moivre $\Rightarrow$
\cos 3\theta + i \sin 3\theta

Equate
\cos 3\theta =
\sin 3\theta =
The answer is not
\cos 3\theta = \cos^3

Harder example

e.g. 3 Find
\sum := \cos \theta + \cos 3\theta + \ldots + \cos(2n + 1)\theta.
A \sum =
But the sum of a
\[ \Sigma' := e \]

Now \[ e^{i2\theta} - 1 = \]

\[ e^{i(2n+3)\theta} \]

\[ \therefore \Sigma' = \]
\[ \Sigma = \]

Complex loci

If we represent \( z = a + bi \) as
then $z$—

**Note:** For vector $v$ as in picture
1. $|v| = $
2. Dirn of

see MATLAB geom2.m file

**Triangle Inequality** For $u, v$

$|u - w|$

**e.g. 4** Sketch $S := \{ z \in \mathbb{C} \mid iz = A \}$