Lecture 3: Computing Powers. Extracting roots.

Aim Lecture Find simple methods to

See usefulness of

Powers via polar form

e.g. 1 Find $(\sqrt{3}-i)^{100}$

Dumb method

Good Method: Write $z = \sqrt{3} - i$ in

$|z| =$

$\operatorname{Arg} z =$

$\therefore z =$

$z^{100} = 2^{100}e$

$=$

as $e^{48\pi i/3} =$

$= 2^{100}($

$= -2^{99}$

n-th roots via polar form

e.g. 2 Find all cube roots of $8i$.

A Suppose $z^3 = 8i, z = r$

Polar form of $8i =$

$z^3 = r$

Equate moduli:

Equate arg:

N.B. $-\pi < \theta \leq \pi \Longrightarrow$

Hence, $3\theta =$

$\therefore \theta =$

So the three cube roots are

$z =$

Let's plot the 3 cube roots

Roots of unity

Method in e.g. 2 gives

Thm 1 The n-th roots of unity i.e.

$z =$

Check: $z = e^{i2k\pi/n} \implies$

Thm 2 Let $\omega :=$

If z_0 is an n-th root of $\alpha \in \mathbb{C}$ then the

$z = z_0,$

Proof: $(z_0\omega^i)$

$\implies z_0\omega^i$

Conversely, if $z_1^n =$

$(\frac{z_1}{z_0})$

Thm 1 $\implies$

so $z_1 =$

Multn & rotation

Shows multn by $e^{i\phi}$

Gives geom interpretation of thm 2. If z_0 is an n-th root of α then

n-th roots are equally

Square roots via cart form

e.g. 3 Solve $z^2 = -5 + 12i$

A Let $z = a+$

z^2

Equate real & imag parts

Solve simultaneously by guessing or elim a or better still use

$|z^2|$

$\therefore 2a^2 =$

$b =$

So square roots are

Quadratic formula

$az^2 + bz + c = 0, \quad a, b, c \in \mathbb{C}$

has solns

e.g. 4 Solve $z^2 + (-4 + i)z + (5 - 5i) = 0.$

discriminant =

From e.g. 3 we see