
**Aim Lecture** Interpret complex numbers

Leads to useful

Argand diagram. Polar form.

Represent $z = a + bi \in \mathbb{C}$ by
Polar coords on plane suggests

**Defn** The modulus of $z = a + |z| :=$
The argument of $z \neq$

so $\tan \theta =$

**e.g. 1**

$| -3 - 4i| =$

$\text{Arg} -3-$

**e.g. 2** $|\bar{z}| =$

**Polar form** For $r = |z|, \theta$
trig \implies z = 

This is called the

\textbf{Euler’s formula for complex exp fn}

\textbf{Lemma} \ (\cos \theta + i \sin \theta)(\cos

\textbf{Proof} \quad \text{LHS} = (\cos \theta \cos \phi - 

+ i(\sin \theta \cos \phi

\textbf{Euler’s Formula} \quad \text{For } \theta \in

More gen for \( a, b \)

\( e^{a+bi} := \)

N.B. \( |e^{a+bi}| = \)
Properties of exponential fn

**Q** Why is Euler’s defn

One **A** Have desirable

**Facts** 1. It recovers real exp fn when $z$
2. $e^{z+}$
3. $(e^z)^{-1} =$
4. For $n \in \mathbb{Z}$, $(e^z)^n =$

Proof: 1. easy.
2. Write $z = a + bi$, $z' = a'$

$LHS = e^{a+bi+}$
\[ e^a e^{a'} (\cos (b + \text{lemma}) = e^a e^{a'} (\cos b ) \]

= 

3. \( e^{-z} e \)

4. For \( n \geq 1 \) it follows by

Clear for \( n = \)

For \( n < 0, \)

An immediate corollary is

**De Moivre’s Thm**

\((\cos \theta + \)

Proof: LHS = \((e^i \)

**Fact 1.** For \( n \in \mathbb{Z} \)

\( e^i(\theta + 2n \)
2. \( e^{i\theta} = e^{i\theta'} \implies \theta - \theta' \)

Proof: 1. holds as \( \cos, \sin \)

2. \( e^{i\theta} = e^{i\theta'} \implies e^{i(\theta)} \)

\( \implies \cos(\theta) \)

**Products in polar form**

For \( r, \theta \in \mathbb{R} \),

\( r \cos \)

This is alternate polar

**Consequences** For \( z, w \in \)

1. \( |\frac{z}{w}| = \)

2. \( \text{Arg } zw = \)

3. \( |z^n| = \)
4. Arg $z^n$

**Proof** half of 1 & 2 only. Others sim.

Let $z = re^{i\theta}$, $w = \quad \frac{z}{w} = \quad$

This is polar

Hence, $|\frac{z}{w}| = \quad$ Arg

**e.g. 4** Let $z = -1 + i, w = 3e^{-2i}$

$|zw| = \quad$

Arg $zw$