Lecture 2: Polar form. Complex exp fn.

Aim Lecture Interpret complex numbers

Leads to useful

Argand diagram. Polar form.

Represent $z = a + bi \in \mathbb{C}$ by

Polar coords on plane suggests

Defn The modulus of $z = a+$

$|z| :=$

The argument of $z(\neq$

so $\tan\theta =$

e.g. 1

$|-3-4i| =$

$\operatorname{Arg} -3-$

e.g. 2 $|\bar{z}| =$

Polar form For $r = |z|, \theta$

trig $\Longrightarrow z =$

This is called the

Euler's formula for complex exp fn

Lemma $(\cos\theta + i\sin\theta)(\cos$

Proof LHS $= (\cos\theta\cos\phi -$

$+i(\sin\theta\cos\phi$

Euler's Formula For $\theta \in$

More gen for a, b

$e^{a+bi} :=$

N.B. $|e^{a+bi}| =$

Arg

e.g. 3 $e^{i\pi/2}$

Properties of exponential fn

Q Why is Euler's defn

One **A** Have desirable

Facts 1. It recovers real exp fn when z

2. e^{z+}

3. $(e^z)^{-1} =$

4. For $n \in \mathbb{Z}$, $(e^z)^n =$

Proof: 1. easy.

2. Write $z = a + bi, z' = a'$

LHS $= e^{a+bi+}$

$= e^a e^{a'}(\cos(b+$

$\overset{\text{lemma}}{=} e^a e^{a'}(\cos b$

$=$

3. $e^{-z}e$

4. For $n \geq 1$ it follows by

Clear for $n =$

For $n < 0$,

An immediate corollary is

De Moivre's Thm

$(\cos\theta +$

Proof: LHS $= (e^i$

Fact 1. For $n \in \mathbb{Z}$

$e^{i(\theta+2n}$

2. $e^{i\theta} = e^{i\theta'} \implies \theta - \theta'$

Proof: 1. holds as cos, sin

2. $e^{i\theta} = e^{i\theta'} \implies e^{i(\theta}$

$\implies \cos(\theta$

Products in polar form

For $r, \theta \in \mathbb{R}$,

$r \cos$

This is alternate polar

Consequences For $z, w \in$

1. $|\frac{z}{w}| =$

2. $\operatorname{Arg} zw =$

3. $|z^n| =$

4. $\text{Arg}\, z^n$

Proof half of 1 & 2 only. Others sim.

Let $z = re^{i\theta}, w =$

$\frac{z}{w} =$

This is polar

Hence, $|\frac{z}{w}| =$

Arg

e.g. 4 Let $z = -1 + i, w = 3e^{-2i}$

$|zw| =$

$\text{Arg}\, zw$