Lecture 2: Subspaces

Aim of Lecture: Solns to linear eqns are typically points, lines planes etc in $\mathbb{R}^n$.

These are special subsets e.g. they are linear, not curved. This lecture introduces

Description(Subspace) Let V be a vector space / field $\mathbb{F}$. A subset $W \subseteq V$ is a subspace if when endowed

In this case write

Say W is a proper

Counter-Examples

Counter e.g. 1 $V = \mathbb{R}^2$

$W = \{(x, y) | x^2 + y^2 = 1\}$.

Counter e.g. 2 $V = \mathbb{R}^2$

$W = x$ and y axes.

Test for Subspaces

For examples of subspaces use

Subspace Thm-Defn Let V be a vector space / field $\mathbb{F}$. A subset $W \subseteq V$ is a subspace if

a.

& b.

i.e.

& c.

In this case, W is a vector space with addn & scalar multn the same as that in V.

Proof: b),c) show addn & scalar multn laws on V give corresponding

Now just check axioms for V give those e.g.

Propn The zero vector of a vector space V is the zero vector of any subspace W.

Proof: Let $\mathbf{0_V}$ be the

Let $\mathbf{w} \in W$. Then $\mathbf{0_V} = \mathbf{w} -$

Examples of Subspaces

E.g. 1 A plane W in $\mathbb{R}^3$ is a subspace iff it goes through the origin.

Proof: If W is a subspace,

Conversely, if W is a plane containing the origin we use subspace criterion to see it's a supbspace. Clearly W is non-

Parametric Form:

check closure under addn:

scalar multn: for $\alpha \in \mathbb{R}$,

Subspace thm-defn

More generally,

Thm The subspaces of $\mathbb{R}^3$ are precisely the following:

No Proof:

E.g. 2 $V = M_{nn}(\mathbb{F})$.

$W =$ subset of

W is a subspace.

Why?

If $A, B \in W$ then,

If $\lambda \in \mathbb{F}$ then,

Subspace thm-defn

E.g. 3 $\mathbb{P}$ = vector space of real polynomials

$\mathbb{P}_k$ = subset of polynomials of degree $\leq k$ is

E.g. 4 X = non-empty set.

$\mathcal{R}[X] :=$

$\mathcal{R}[X]$ is a vector space / $\mathbb{R}$ if define

Consider $\mathcal{R}[\mathbb{R}]$ and subset $\mathcal{C}[\mathbb{R}]$ of

This is a subspace $\because$