This problem set covers material from lectures 9-13.

1. Compute the Betti numbers of the Klein bottle and the projective plane.

2. Show that the relation of being isomorphic in a category is an equivalence relation. (You probably should restrict to the case where the class of objects form a set. These are called small categories.)

3. Show that $\tilde{H}_0 : \text{Simp} \to \text{Ab}$ is a functor.

4. Prove that the figure 8 is not homeomorphic to $S^1$.

5. Let $f : S^2 \to S^2$ be rotation about some axis through angle $\theta$. Compute $\text{deg } f$.

6. (Argument principle in complex analysis). Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function. We consider $S^1 = \{ z \in \mathbb{C} | |z| = 1 \}$ and let $r : \mathbb{C} - 0 \to S^1 : z \mapsto \frac{z}{|z|}$ be the usual retraction. We assume that $f$ has no zeros on $S^1$. Let $h = rf : S^1 \to S^1$. Show that $\text{deg } h$ is the number of zeros (counting multiplicity) of $f$ in the unit disc, which is also of course

$$\frac{1}{2\pi i} \int_{S^1} \frac{f'(z)}{f(z)} \, dz.$$