1. Let $A = \{a, b, c, d\}$ be ordered in alphabetical order.

(a) Which of the following defines a simplicial complex:

- $K = \{a, b, c, d, ab, bc, bd\}$
- $K' = \{a, b, c, d, ab, bc, abc\}$

(b) For each of the simplicial complexes in part (a), draw a figure which is homeomorphic to the polytope of that simplicial complex.

2. Show that the following sequence of abelian groups is a chain complex and compute all of its homology groups.

$$0 \rightarrow \mathbb{Z} \overset{\phi}{\rightarrow} \mathbb{Z} \oplus \mathbb{Z} \overset{\psi}{\rightarrow} \mathbb{Z} \rightarrow 0$$

where $\phi(n) = (n, -2n)$, $\psi(m, n) = 2m + n$.

3. Consider the triangulation $\theta : |K| \rightarrow X$ of the Klein bottle given by the labelled surface diagram below.

Compute the homology groups $H_p(K)$. Hint: Follow the example of the torus in lecture 5.

4. Let $s_\bullet$ be a chain homotopy between chain maps $f_{0\bullet}, f_{1\bullet} : C_\bullet \rightarrow C'_\bullet$.

Let $g_\bullet : C'_\bullet \rightarrow C''_\bullet$ be another chain map. Prove that $g_\bullet f_{0\bullet}, g_\bullet f_{1\bullet}$ are also chain homotopic, i.e. there is a chain homotopy between them.
5. Let \( \theta : \vert K \vert \rightarrow X \) be a triangulation of a topological space. Consider the quotient space \( Y = (X \times I) / \sim \) where \( \sim \) is the equivalence relation generated by \((x, 1) \sim (x', 1)\) for all \( x, x' \in X \). Find a triangulation for \( Y \).