1. A cake is divided into 6 equal sectors. In the middle of each candle is placed a red, green or blue candle. How many essentially different ways are there of doing this?

2. In this question, we consider the group $G$ generated by the reflections

$$
\tau_1 := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \tau_2 := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
$$

We let $G$ act on the $xy$-plane in the usual way (that is, by matrix multiplication).

(a) Are $\tau_1, \tau_2$ symmetries of the square with sides $x = \pm 1, y = \pm 1$?

(b) Show that $G = \langle \tau_1, \tau_2 \rangle$ is isomorphic to the dihedral group $D_4$ of order 8. (Hint: what is the rotation $\tau_1 \tau_2$).

(c) Let $G$ act on $S := \mathbb{R}^2$ by matrix multiplication, i.e. $g.v := gv$ for $v \in \mathbb{R}^2$. Find the fixed point set $S^G$.

(d) Let $v := (\frac{1}{2})$. Find the orbit of $v$.

(e) Find $H < G$ such that $G.v \simeq G/H$.

3. Describe explicitly, the 24 elements in the rotational symmetry group of the cube. (Hint: Use the fact that the poles which give the axes of rotation correspond to faces, edges and vertices of the cube).

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