

CLASS TEST 2

ANSWER ALL QUESTIONS

THE MARKS FOR QUESTIONS ARE AS SHOWN

TIME ALLOWED - FORTY-FIVE MINUTES

WRITE YOUR TUTORIAL GROUP ON THE FRONT OF THE BOOK.

1. (**6 marks**) A sheet of metal of constant surface density $\delta(x, y) = K$, K constant, is bounded by the straight lines $y = x$, $y = -x$ and the circle $x^2 + y^2 = 4$ for $x \geq 0$. Carefully sketch the sheet in the (x, y) plane. Find the moment of inertia of the sheet about the y axis. (HINT: use polar coordinates).
2. (**4 marks**) Sketch the region of integration for the integral,

$$\int_{-1}^0 \int_{-y}^1 \sin(x^2) dx dy.$$

Hence express the integral with the order of integration reversed and then evaluate it.

3. (**4 marks**) A surface in 3 dimensions is given by

$$\mathbf{x}^T A \mathbf{x} = 0$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

and A is a symmetric 3×3 matrix. You are given that the eigenvalues of A are 1, 2 and -3 . Write down a possible principal axes form of the surface. What shape does this quadric surface represent? Sketch it in the principal axes frame (X, Y, Z) .

4. (**6 marks**) Find the general solution of the differential equation

$$\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 5e^{2x}$$

with $y(0) = 1, y'(0) = 2$.