MATH2019 **Session 2, 2004**

CLASS TEST 1, 2004

ANSWER ALL QUESTIONS
TIME ALLOWED - FORTY FIVE MINUTES
WRITE YOUR **TUTOR'S NAME AND TUTORIAL TIME**
ON THE FRONT OF THE EXAM BOOK.

Leibniz Rule for Differentiation of Integrals

$$\frac{d}{dx}\int_u^v f(x,t)dt = \int_u^v \frac{\partial f}{\partial x}dt + f(x,v)\frac{dv}{dx} - f(x,u)\frac{du}{dx}.$$

1. Consider the scalar field $f(x,y,z) = x^2+y^2+z^2$. Find the directional derviative at the point $P(1,-1,2)$ in the direction of the vector $\mathbf{u} = (1,2,3)$. Also find the tangent plane to the sphere $x^2+y^2+z^2 = 6$ at the point P (**5 marks**).

2. Evaluate using Leibniz rule (**5 marks**):

$$\frac{d}{d\alpha}\int_{\sqrt{\alpha}}^{\alpha} \frac{\cos\alpha x}{x}\,dx$$

3. Use the method of Lagrange multipliers to find the point on the plane

$$3x+4y-z = 26$$

closest to the origin (**5 marks**).

4. The kinetic energy T of a particle of mass m and velocity v is given by

$$T = \frac{1}{2}mv^2.$$

If T has decreased by 5% while v has increased by 2%, estimate the corresponding percentage change in the mass m (indicate whether it is an increase or a decrease). (**5 marks**).

5. Find all the stationary points of the function

$$f(x,y) = \frac{1}{2}x^2 + xy + x + \frac{1}{3}y^3 - y$$

and classify each as a local maximum, local minimum or saddle point. Also give the function values at the points (**5 marks**).

P.T.O.

6. This question contains 5 multiple choice subquestions. Select the answer that best fits the situation by writing the letter to its left in your answer booklet. (Select only one answer in each case.) (**5 marks**)

(i) Let **F** be a vector field representing the velocity of a gas. The expression div$\mathbf{F}(x, y, z)$ represents:

(a) the tendancy of the gas to spread away from the pt (x, y, z)

(b) the tendancy of the gas to swirl around the pt (x, y, z)

(c) the temperature of the gas at the pt (x, y, z)

(d) the volume of the gas.

(ii) For $z = f(x, y)$, the partial dervative $\partial f/\partial y$ evaluated at a point (a, b) gives:

(a) the rate of change of f measured in the direction of the positive x-axis

(b) the rate of change of f measured in the direction of the positive y-axis

(c) the gradient vector

(d) the rate of change of f measured in any direction.

(iii) Let **V** be a vector field representing the velocity of a fluid in a pipe. The expression curl $\mathbf{V}(x, y, z)$ represents:

(a) the tendancy of the fluid to spread away from the point (x, y, z)

(b) the tendancy of the fluid to swirl around the point (x, y, z)

(c) the viscosity of the fluid at the pt (x, y, z)

(d) the size of the pipe at the pt (x, y, z).

(iv) For $w = f(x, y, z)$, $x = x(r, s)$, $y = y(r, s)$, $z = z(r, s)$ the deritative $\partial f/\partial r$ is:

(a) $\frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$

(b) $\frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r}$

(c) $\frac{\partial f}{\partial x}\frac{\partial x}{\partial r}$

(d) $\frac{\partial f}{\partial r}\frac{\partial r}{\partial x}$.

(v) If **F** is a vector field then:

(a) div **F** is a vector field and curl **F** is a vector field

(b) only div **F** is a vector field

(c) only curl **F** is a vector field

(d) div **F** and curl **F** are not vector fields.

END OF PAPER