MATH2019 **Session 2, 2001**

CLASS TEST 1

ANSWER ALL QUESTIONS
THE MARKS FOR QUESTIONS ARE AS SHOWN
TIME ALLOWED - FORTY FIVE MINUTES
WRITE YOUR TUTORIAL GROUP NUMBER ON THE FRONT OF THE EXAM BOOK (SEE LIST ON BACK).

Leibniz Rule for Differentiation of Integrals

$$\frac{d}{dx}\int_u^v f(x,t)dt = \int_u^v \frac{\partial f}{\partial x}dt + f(x,v)\frac{dv}{dx} - f(x,u)\frac{du}{dx}$$

Multivariable Taylor Series

$$\begin{aligned} f(x,y) &= f(a,b) + (x-a)\frac{\partial f}{\partial x}(a,b) + (y-b)\frac{\partial f}{\partial y}(a,b) \\ &\quad + \frac{1}{2!}\left((x-a)^2\frac{\partial^2 f}{\partial x^2}(a,b) + 2(x-a)(y-b)\frac{\partial^2 f}{\partial x \partial y}(a,b) + (y-b)^2\frac{\partial^2 f}{\partial y^2}(a,b)\right) + \cdots \end{aligned}$$

1. Expand $f(x,y) = e^x \sin y$ about $(1,0)$ up to and including second-order terms (**3 marks**).

2. Evaluate using Leibniz rule (**5 marks**):

$$\frac{d}{d\alpha}\int_{\sqrt{\alpha}}^{\alpha} \frac{\cos \alpha x}{x}\, dx$$

3. Suppose V is calculated from the formula $V = R^2 H$. If $R = 10.0$ cm and $H = 3.0$ cm to the nearest millimetre, estimate the maximum possible error in the computed value of V (**4 marks**).

4. Find all the stationary points of the function

$$f(x,y) = \frac{1}{2}x^2 + xy + x + \frac{1}{3}y^3 - y$$

and classify each as a local maximum, local minimum or saddle point. Also give the function values at the points (**6 marks**).

5. Given the scalar field

$$\phi(x,y,z) = ze^{xy},$$

calculate:

   (a) The directional derivative of ϕ at the point $\mathrm{P}(0,2,\frac{1}{2})$ in the direction of the vector $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ (**4 marks**).

   (b) the tangent plane to the surface $\phi(x,y,z) = \frac{1}{2}$ at the point P (**3 marks**).

P.T.O.