Math 5215 W14

1b) $y(t+1) - t\,y(t) = -3^t$

(not in form) $y^\Delta = -p(t)y^\sigma + f(t)$

rearrange above,

$$y(t) = \frac{3^t + y(t+1)}{t}$$

subs

So $$y(t+1) = \frac{3^{t+1} + y(t+2)}{t+1}$$

$$\Rightarrow \quad y(t) = \frac{3^t + \dfrac{3^{t+1} + y(t+2)}{t+1}}{t}$$

Continuing in this fashion, obtain.

$$y(t) = \frac{3^t + \dfrac{3^{t+1} + \dfrac{3^{t+2} + \frac{3^{t+3}}{t+3} \cdots}{t+2}}{t+1}}{t}$$

and formally dividing throughout, obtain

$$y(t) = \frac{3^t}{t} + \frac{3^{t+1}}{t(t+1)} + \frac{3^{t+2}}{t(t+1)(t+2)} + \cdots$$
$$= \sum_{k=0}^{\infty-1} \frac{3^{t+k}}{t(t+1)\cdots(t+k)}$$

This sum converges (ratio test), at pts $t \neq 0, -1, -2, \ldots$

General solution

$$y(t) = \sum_{k=0}^{\infty} \frac{3^{t+k}}{t(t+1)\cdots(t+k)} + c(t) \qquad \text{where} \quad \Delta c(t) = 0 \ \forall t$$

1 C, (i) The summation by parts formula gives

$$(*) \qquad \sum a_n(\Delta b_n) = a_n b_n - \sum (\Delta a_n) b_{n+1}$$

Now we also know

$$\sum y_n = \sum_{k=m}^{n-1} y_k + C \qquad (m<n)$$

So subs above eq^n into $(*)$ gives us

$$\sum_{k=m}^{n-1} a_k(\Delta b_k) = a_n b_n - \sum_{k=m}^{n-1} (\Delta a_k) b_{k+1} + C \quad -(**)$$

find C For $n = m+1$ in $(**)$ we obtain $C = a_m b_m$

Hence $(**)$ becomes

$$\sum_{k=m}^{n-1} a_k(\Delta b_k) = [a_k b_k]_m^n - \sum_{k=m}^{n-1} (\Delta a_k) b_{k+1}$$

(ii) $\sum k^2 = \sum (k^{\underline{2}} - k^{\underline{1}})$ $\quad \begin{cases} k^{\underline{2}} = k(k+1) \\ k^{\underline{1}} = k \end{cases}$

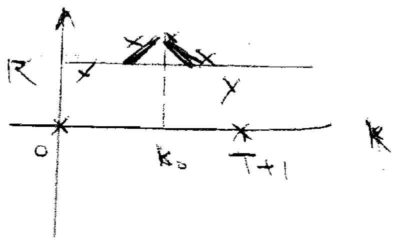
$$= \frac{1}{3}k^3 - \frac{1}{2}k^2 + C$$

1d, Argue by contradiction

Let $r(k) = [x(k)]^2 - R^2$, $k=0,\dots,T+1$
where $x(k)$ is a sol^{n} to (1), (2)

Let $k_0 \in \{0,\dots,T+1\}$ $\ni$

$$r(k_0) = \max_{k\in\{0,\dots,T+1\}} r(k) \geq 0$$



Obviously (from (2)) $k_0 \neq 0, T+1$

Therefore $k_0 \in \{1,\dots,T\}$

$$\Delta r(k_0) \leq 0, \quad \Delta r(k_0-1) \geq 0$$

$$\Delta^2 r(k_0-1) \leq 0$$

$$0 \geq \Delta^2 r(k_0-1) = \Delta^2 [x(k)^2 - R^2] \Big|_{k=k_0-1}$$

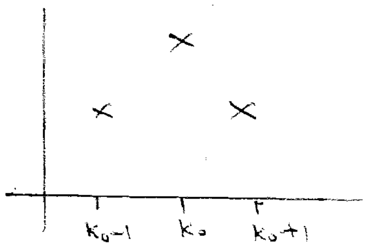
$$= 2x(k_0)\Delta^2 x(k_0-1) + (\Delta x(k_0-1))^2 + (\Delta x(k_0))^2$$

(prod rule twice)

$$\geq 2x(k_0)\Delta^2 x(k_0-1)$$
$$= 2x(k_0)\, f(k_0, x k_0) > 0 \quad \text{by assumption}$$

and hence we reach a contradiction and we conclude

$$|x(k)| < R, \quad k=0,\dots,T+1$$



"discrete maximum principle"

$r(k_0) \geq r(k_0+1)$
$r(k_0) \geq r(k_0-1)$

ie. $\Delta r(k_0) \leq 0$
$\Delta r(k_0-1) \geq 0$

$\Delta^2 r(k_0-1) \leq 0$

Q2 a extend, unify, ~~discretise~~ discrete ideas from contin & discrete calculus

b, (ii) $\mathbb{R}$, $h\mathbb{Z}$, Cantor Set

c, Bohr + Peterson Thm 1.16 (ii) pg. 5

d, Since $\mathbb{T}$ is isolated

$$y^{\Delta}(t) = \frac{y(\sigma(t)) - y(t)}{\mu(t)} \quad \forall t \in \mathbb{T}^{k}.$$

$$(f(t)g(t))^{\Delta} = \frac{f(\sigma(t))g(\sigma(t)) - f(t)g(t)}{\mu(t)}$$

$$= \left(\frac{f(\sigma(t)) - f(t)}{\mu(t)}\right) g(\sigma(t)) + \frac{f(t)g(\sigma(t)}{\mu(t)}$$

2e, $(fg)^\Delta = f^\Delta g + f^\sigma g^\Delta$

but, $(fg)^{\Delta\Delta} = (f^\Delta g + f^\sigma g^\Delta)^\Delta$

may not exist as $(f^\sigma)^\Delta$ may not exist.

$(f^\sigma)^\Delta$ will exist when σ^Δ exist.

3 a, Bohner + Peterson $Th^{=}$ 1.74 pg. 27

c, (i) $\int_1^\infty \frac{1}{t^2}\,\Delta t$, $\mathbb{T} = \{q^0, q^1, q^2, \dots\}$, $q > 1$

$= \lim\limits_{b\to\infty}\left[\int_1^b \frac{1}{t^2}\,\Delta t\right]$ here $\sigma(t) = qt$.

$= \lim\limits_{b\to\infty}\left[\int_1^b \frac{q}{(qt)t}\,\Delta t\right]$ we know $\left(\frac{1}{t}\right)^\Delta = \frac{-1}{t\sigma(t)}$

$= \lim\limits_{b\to\infty}\left[\int_1^b \frac{-q}{\sigma(t)t}\,\Delta t\right]$

$= \lim\limits_{b\to\infty}\left\{\left[\frac{q}{t}\right]_1^b\right\}$

$= q$.