MATH5215 LECTURE 10/5/2005

Last time we solved the homogeneous problem $y^{\Delta\Delta}+2y^{\Delta}-3y=0$. What about the inhomogeneous case?

Consider $y^{\Delta\Delta}+2y^{\Delta}-3y=f(t)$. If y_h is a solution to the homogeneous problem and if y_p is a particular solution to the inhomogeneous problem then $y=y_h+y_p$.

(1) $f(t)\equiv 1$. What is y_p? Try $y_p=K$, K a constant. For $y_p=K$, the homogeneous problem becomes $y_p^{\Delta\Delta}+2y_p^{\Delta}-3y_p=0+0-3K=1$
$\Rightarrow K=-\frac{1}{3}$,
$\Rightarrow y=y_h+y_p=\alpha e_{-3}(t,0)+\beta e(t,0)-\frac{1}{3}$, α and β constants.

(2) $f(t)=t$. Try $y_p=At+B$, where A and B are constants. $y_p^{\Delta}=A$, $y_p^{\Delta\Delta}=0$, so $y_p^{\Delta\Delta}+2y_p^{\Delta}-3y_p=0+2A-3At-3B=t$
$\Rightarrow 2A-3B=0$
$\Rightarrow -3A=1$,
so $A=-\frac{1}{3}$, $B=\frac{2}{9}$.
$\Rightarrow y=y_h+y_p=\alpha e_{-3}(t,0)+\beta e(t,0)-\frac{1}{3}t+\frac{2}{9}$

(3) $f(t)=t^2$ (open case) Try $y_p=At^2+Bt+C$, where A,B,C are constants.
$y_p^{\Delta}=A[t+\sigma(t)]+B$
$y_p^{\Delta\Delta}=??$ (it may not exist because σ may not be delta differentiable)

(4) $f(t)=e_2(t,0)$. Try $y_p=Ke_2(t,0)$, where K is a constant.
$y_p^{\Delta}=2Ke_2(t,0)$, $y_p^{\Delta\Delta}=4Ke_2(t,0)$,
$y_p^{\Delta\Delta}+2y_p^{\Delta}-3y_p=4Ke_2(t,0)+4Ke_2(t,0)-3Ke_2(t,0)=e_2(t,0)$
$\Rightarrow 5K=1$, so $K=\frac{1}{5}$,
$\Rightarrow y=y_h+y_p=\alpha e_{-3}(t,0)+\beta e(t,0)+\frac{1}{5}e_2(t,0)$

Consider the system

$$\begin{cases} x^{\Delta}=2x+y & (1)\\ y^{\Delta}=-5x+4y & (2)\\ x(0)=1 & (3)\\ y(0)=2 & (4)\end{cases}$$

$$\begin{aligned}(2) \Rightarrow y^{\Delta\Delta} &= -5x^{\Delta} - 4y^{\Delta} \\ &= -5(2x+y) - 4y^{\Delta} \\ &= -10x - 5y - 4y^{\Delta} \\ &= -10(\frac{-4y - y^{\Delta}}{5}) - 5y - 4y^{\Delta} \\ &= 8y + 2y^{\Delta} - 5y - 4y^{\Delta}\end{aligned}$$

$\Rightarrow y^{\Delta\Delta} + 2y^{\Delta} - 3y = 0$, which has solution $y = Ae_{-3}(t,0) - Be(t,0)$, where A and B are constants.

$$\begin{aligned}(2) \Rightarrow x &= -\frac{1}{5}(y^{\Delta} + 4y) \\ &= -\frac{1}{5}[-3Ae_{-3}(t,0) + Be(t,0) + 4Ae_{-3}(t,0) + 4Be(t,0)] \\ &= -\frac{1}{5}[Ae_{-3}(t,0) - Be(t,0)] \\ &= -\frac{1}{5}Ae_{-3}(t,0) - Be(t,0)\end{aligned}$$

Now we find A and B:

$$\begin{aligned}x(0) = 1 &= -\frac{1}{5}Ae_{-3}(0,0) - Be(0,0) \\ &= -\frac{A}{5} - B \\ y(0) = 2 &= A + B\end{aligned}$$

$\Rightarrow A = \frac{15}{4}$, $B = -\frac{7}{4}$, so our full solution is:

$$\begin{aligned}x(t) &= -\frac{1}{5} \cdot \frac{15}{4}e_{-3}(t,0) + \frac{7}{4}e(t,0) \\ &= -\frac{3}{4}e_{-3}(t,0) + \frac{7}{4}e(t,0)\end{aligned}$$

$$y(t) = \frac{15}{4}e_{-3}(t,0) - \frac{7}{4}e(t,0)$$