1. (6 marks) A sheet of metal of constant surface density \( \delta(x, y) = K \), \( K \) constant, is bounded by the straight lines \( y = x \), \( y = -x \) and the circle \( x^2 + y^2 = 4 \) for \( x \geq 0 \). Carefully sketch the sheet in the \((x, y)\) plane. Find the moment of inertia of the sheet about the \( y \) axis. (HINT: use polar coordinates).

2. (4 marks) Sketch the region of integration for the integral,

\[
\int_{-1}^{0} \int_{-y}^{1} \sin(x^2) \, dx \, dy.
\]

Hence express the integral with the order of integration reversed and then evaluate it.

3. (4 marks) A surface in 3 dimensions is given by

\[ x^T A x = 0 \]

where

\[
x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}
\]

and \( A \) is a symmetric \( 3 \times 3 \) matrix. You are given that the eigenvalues of \( A \) are 1,2 and \(-3\). Write down a possible principal axes form of the surface. What shape does this quadric surface represent? Sketch it in the principal axes frame \((X, Y, Z)\).

4. (6 marks) Find the general solution of the differential equation

\[
\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 9y = 5e^{2x}
\]

with \( y(0) = 1, y'(0) = 2 \).