MATH5215, Some Questions Involving $e_p(t, t_0)$

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1. Consider the following dynamic IVP on arbitrary $\mathbb{T}$

$$y^\Delta = p(t)y, \quad p \in \mathcal{R}, \tag{0.1}$$
$$y(t_0) = 1, \quad t_0 \in \mathbb{T}. \tag{0.2}$$

   Show that the only solution to (0.1), (0.2) is $e_p(t, t_0)$. (Hint: Let y be a solution to the IVP and show

$$\left[\frac{y(t)}{e_p(t, t_0)}\right]^\Delta = 0$$

   so that $\frac{y(t)}{e_p(t,t_0)} = constant$.)

2. Let $\alpha \in \mathcal{R}$ be a constant. Find $e_\alpha(t, t_0)$ for

$$t, t_0 \in \mathbb{T} = q^{\mathbb{N}_0} = \{q^0, q^1, q^2, \ldots\}$$

   where $q > 1$ and constant. What is $e_\alpha(t, 1)$ for $\mathbb{T} = 2^{\mathbb{N}_0}$?

3. Consider the following dynamic IVP on arbitrary $\mathbb{T}$

$$y^\Delta = p(t)y, \quad p \in \mathcal{R}, \tag{0.3}$$
$$y(t_0) = y_0, \quad t_0 \in \mathbb{T}, \ y_0 \in \mathbb{R}. \tag{0.4}$$

   Under which conditions will solutions to (0.3), (0.4): oscillate; be non-oscillatory; be strictly positive; be strictly negative?

4. Let $N(t)$ be the number of plants of a certain species at time t. During the months of April until September, N grows exponentially according to $N' = N$. At the beginning of October, all plants suddenly die out, but the seeds remain in the ground and start growing again when they germinate in the following April with N now being doubled. The appropriate time scale to model N on is given by

$$\mathbb{T} = \cup_{k=0}^{\infty}[2k, 2k+1].$$

   Explain why the model leads to the dynamic equation $N^\Delta = N$. If there is originally one plant then solve the dynamic IVP for N.

5. What is $e_\alpha(t, t_0)$ for $\mathbb{T} = h\mathbb{Z}$, where $\alpha \in \mathcal{R}$ and constant?

6. Show that the solution to the dynamic (nonhomogeneous) IVP

$$\begin{aligned} y^\Delta &= -p(t)y^\sigma + f(t), \quad p \in \mathcal{R}, \quad f \in C_{rd}, \\ y(t_0) &= y_0, \quad t_0 \in \mathbb{T}, \quad y_0 \in \mathbb{R}, \end{aligned}$$

   is

$$y(t) = \frac{y_0 + \int_{t_0}^{t} e_p(s, t_0) f(s) \Delta s}{e_p(t, t_0)}.$$

7. Solve

$$\begin{aligned} y^\Delta &= -p(t)y^\sigma + 1/e_p(t, t_0), \quad p \in \mathcal{R}, \\ y(t_0) &= 1, \quad t_0 \in \mathbb{T}. \end{aligned}$$