

MATH5215, Some Questions Involving the Delta-Integral

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1. Can you provide some examples of various time scales ($\mathbb{T} \neq \mathbb{R}$) such that σ satisfies:
 - (i) σ is continuous on \mathbb{T}
 - (ii) σ is not continuous on the whole of \mathbb{T}
 - (iii) σ is right-dense (rd) continuous on \mathbb{T}
 - (iv) σ is regulated on \mathbb{T} but not rd-continuous on \mathbb{T} ?

2. Let $x : \mathbb{T} \rightarrow \mathbb{R}$. Show that if x is continuous on \mathbb{T} then x is also rd-continuous on \mathbb{T} .

3. Consider

$$f(t) = \begin{cases} 0, & t \in \mathbb{N} \\ t, & \text{otherwise} \end{cases}$$

Is f regulated on $\mathbb{T}_1 = \mathbb{N}_0 \cup \{1 - 1/n : n \in \mathbb{N}\}$. Is it rd-continuous on \mathbb{T}_1 ? How about the time scale

$$\mathbb{T}_2 = \mathbb{T}_1 \cup \{1 + 1/n : n \in \mathbb{N}\}?$$

4. Prove the following theorem. Every rd-continuous function has an anti(delta)derivative.
5. Let \mathbb{T} be arbitrary and let $\alpha \in \mathcal{R}$ be a constant. Find

$$\int_a^b (\mu(t) + t)e_\alpha(t, 0)\Delta t, \quad a, b \in \mathbb{T}.$$

6. If $f \in C_{rd}$ and $t \in \mathbb{T}^\kappa$ then show that

$$\int_t^{\sigma(t)} f(s)\Delta s = \mu(t)f(t).$$

Hence show that if $[a, b]_{\mathbb{T}}$ consists of only isolated points and $a, b \in \mathbb{T}$ then

$$\int_a^b f(t)\Delta t = \sum_{t \in [a, b]_{\mathbb{T}}} \mu(t)f(t).$$