

MATH5215, Some Questions Involving the Delta-Derivative

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1. Prove the following theorem.

Theorem 0.1 *Assume that $g : \mathbb{T} \rightarrow \mathbb{R}^n$ and let $t \in \mathbb{T}^\kappa$.*

(i) If g is d -differentiable at t then g is continuous at t .

(ii) If g is continuous at t and t is right-scattered then g is d -differentiable at t with

$$g^\Delta(t) = \frac{g(\sigma(t)) - g(t)}{\sigma(t) - t}.$$

(iii) If g is d -differentiable and t is right-dense then

$$g^\Delta(t) = \lim_{s \rightarrow t} \frac{g(t) - g(s)}{t - s}.$$

(iv) If g is d -differentiable at t then $g(\sigma(t)) = g(t) + \mu(t)g^\Delta(t)$.

Hence show that any function $x : \mathbb{Z} \rightarrow \mathbb{R}$ (here $\text{dom } x = \mathbb{Z}$) is continuous on \mathbb{Z} .

2. Prove the product rule for arbitrary time scales.
3. Give an example of a time scale where (i) σ is not continuous, (ii) σ is not delta-differentiable. Prove your claim.