MATH5215, Some Questions Involving Σ

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Below are some very gentle exercises to get you working with the summation operator Σ.

1. Let $\Delta C(t) = 0$. Show that

$$\sum \cos at = \frac{\sin a(t-1/2)}{2\sin(a/2)} + C(t), \quad a = 2n\pi$$

$$\sum \binom{t+r}{r} = \binom{t+r}{t-1}$$

2. Use summation by parts to compute: $\sum t \sin t$, $\sum t^2 3^t$,

$$\sum \frac{t}{(t+1)(t+2)(t+3)},$$

$$\sum \binom{t}{2}\binom{t}{7}, \quad \sum \binom{t}{2}^2$$

3. Let $z_n = \sum y_n$. Show that

$$\sum_{k=m}^{n-1} y_k = z_n - z_m$$

4. Show that

$$\frac{1}{2} + \sum_{k=1}^{n-1} \cos ak = \frac{\sin a(n-1/2)}{2\sin(a/2)}$$

5. Compute

$$\sum_{k=1}^{8} \frac{1}{(k+1)(k+2)(k+3)}$$

6. Compute

$$\sum_{k=1}^{n-1} \frac{k}{2^k}$$

7. Compute

$$\sum_{k=2}^{n-1} k^2(k-1)$$

8. There is a special kind of summation based on

$$\Delta^n y(t) = \sum_{k=0}^{n} (-1)^k \left(\begin{array}{c} n \\ k \end{array} \right) y(t+n-k).$$

Use induction on i and summation by parts to show that

$$\sum_{n=i}^{\infty} \left(\begin{array}{c} n \\ i \end{array} \right) \left(\frac{1}{2} \right)^{n+1} = 1$$

for $i = 0, \ldots, \infty$.