

MATH5215, Some Questions Involving Δ

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Below are some very gentle exercises to get you working with the difference operator Δ .

1. Prove the quotient rule for differences
2. Show that for any constant a , $\Delta \sin at = 2 \sin \frac{a}{2} \cos a(t + 1/2)$
3. Show that for any constant a , $\Delta \sinh at = 2 \sinh \frac{a}{2} \cosh a(t + 1/2)$
4. Compute $\Delta(3^t \cos t)$ by two different methods
5. Does the falling factorial power satisfy $t^r t^s = t^{r+s}$?
6. The binomial coefficient $\binom{t}{r}$ satisfies many useful identities, including:

$$\binom{t}{r} = \binom{t}{t-r} \quad (\text{symmetry});$$

$$\binom{t}{r} = \frac{t}{r} \binom{t-1}{r-1} \quad (\text{moving out of parentheses});$$

$$\binom{t}{r} = \binom{t-1}{r} + \binom{t-1}{r-1} \quad (\text{addition formula}).$$

Use these to prove $\Delta_t \binom{r+t}{r} = \binom{r+t}{t+1}$

7. Find a solution to each of the following difference equations:

$$\begin{aligned} y(t+1) - y(t) &= t^3 + 3^t, \\ y(t+2) - 2y(t+1) + y(t) &= \binom{t}{5}. \end{aligned}$$