MATH5215, Solutions to Some Questions Involving Δ

March 23, 2005

1.

$$\begin{aligned}
\Delta\left(\frac{y(t)}{z(t)}\right) &= \frac{y(t+1)}{z(t+1)} - \frac{y(t)}{z(t)} \\
&= \frac{y(t+1)z(t) - y(t)z(t+1)}{z(t)z(t+1)} \\
&= \frac{z(t)Ez(t) - y(t)Ez(t)}{z(t)Ez(t)}
\end{aligned}$$

2. We use the identity

$$\sin u - \sin v = 2\sin\left[\frac{1}{2}(u-v)\right]\cos\left[\frac{1}{2}(u+v)\right].$$

$$\begin{aligned}
\Delta \sin at &= \sin a(t+1) - \sin at \\
&= 2\sin\left[\frac{1}{2}(a(t+1)-at)\right]\cos\left[\frac{1}{2}(a(t+1)+at)\right] \\
&= 2\sin\frac{a}{2}\cos a(t+1/2)
\end{aligned}$$

3. Similar to the above case

4. (a) From first principles we have

$$\Delta(3^t\cos t) = 3^{t+1}\cos(t+1) - 3^t\cos t = 3^{t+1}(\cos(t+1) - \frac{1}{3}\cos t).$$

(b) From the product rule we have

$$\begin{aligned}
\Delta(3^t\cos t) &= \cos t(\Delta 3^t) + 3^{t+1}(\Delta\cos t) \\
&= 2\cdot 3^t\cos t + 3^{t+1}(-2\sin(1/2)\sin(t+1/2)) \\
&= 3^{t+1}\left[\frac{2}{3}\cos t - 2\sin(1/2)\sin(t+1/2)\right].
\end{aligned}$$

Note that the asnwers in (a) and (b) are equal, as

$$\cos(t+1) = \cos t - 2\sin(1/2)\sin(t+1/2).$$

5. No. For example, try $r = 1 = s$.

6. The symmetry identity gives $\Delta_t \binom{r+t}{r} = \Delta_t \binom{r+t}{t} = \binom{r+t}{t+1}$

7. Let $C(t)$ be such that $\Delta C(t) = 0$ for all t. (a) Notice that

$$\Delta(\frac{1}{4}t^{\underline{4}} + \frac{3^t}{2}) = t^{\underline{3}} + 3^t$$

So $y(t) = \frac{1}{4}t^{\underline{4}} + \frac{3^t}{2} + C(t)$ is the general solution.

(b) Notice that

$$\Delta_t^2 \binom{t}{7} = \binom{t}{5}$$

and so $y(t) = \binom{t}{7} + C(t)$ is the general solution.