# MATH 1251 Mathematics 1B Algebra S2 2008
# Test 2 Version 3B (Yellow)

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1. Clearly $\operatorname{span}\{\mathbf{u},\mathbf{v}\} \subseteq \operatorname{span}\{\mathbf{u},\mathbf{v},\mathbf{w}\}$. Now since $\mathbf{w} \in \operatorname{span}\{\mathbf{u},\mathbf{v}\}$ there exist $\mu_1, \mu_2 \in \mathbb{F}$ such that $\mathbf{w} = \mu_1\mathbf{u} + \mu_2\mathbf{v}$. Now if $\mathbf{x} \in \operatorname{span}\{\mathbf{u},\mathbf{v},\mathbf{w}\}$ then there exist $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{F}$ such that

$$\begin{aligned}\mathbf{x} &= \lambda_1\mathbf{u} + \lambda_2\mathbf{v} + \lambda_3\mathbf{w}\\ &= \lambda_1\mathbf{u} + \lambda_2\mathbf{v} + \lambda_3(\mu_1\mathbf{u} + \mu_2\mathbf{v})\\ &= (\lambda_1 + \lambda_3\mu_1)\mathbf{u} + (\lambda_2 + \lambda_3\mu_2)\mathbf{v} \in \operatorname{span}\{\mathbf{u},\mathbf{v}\}.\end{aligned}$$

   Thus $\operatorname{span}\{\mathbf{u},\mathbf{v}\} \supseteq \operatorname{span}\{\mathbf{u},\mathbf{v},\mathbf{w}\}$ and so equality must hold.

2. (a)
$$\begin{pmatrix} 1 & 2 & -1 & 1\\ -2 & -1 & 3 & 0\\ 3 & 9 & -2 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 & 1\\ 0 & 3 & 1 & 2\\ 0 & 0 & 0 & 0 \end{pmatrix}$$
   First two columns are leading so a basis for the column space is
$$\left\{ \begin{pmatrix} 1\\ -2\\ 3 \end{pmatrix}, \begin{pmatrix} 2\\ -1\\ 9 \end{pmatrix} \right\}$$

   (b)
$$\left\{ \begin{pmatrix} 1\\ -2\\ 3 \end{pmatrix}, \begin{pmatrix} 2\\ -1\\ 9 \end{pmatrix}, \begin{pmatrix} 0\\ 0\\ 1 \end{pmatrix} \right\}$$
   clearly does the job. Plenty of other answers of course, but they all require work, where as this one is obvious. If you don't see why this is obvious ask me!!!! (No-body got this "obvious" answer.)

3. Form an augmented matrix and row reduce without making mistakes. You will get

$$\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$$

If you got anything else, you are wrong - check your working out carefully.

4. $T(X+Y) = A(X+Y)B = AXB + AYB = T(X) + T(Y)$. Also, $T(\lambda X) = A\lambda XB = \lambda AXB = \lambda T(X)$. Thus is is linear. This question has nothing to do with "closure".