MATH 1251 Mathematics 1B Algebra S2 2008
Test 2 Version 3A (Green)

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1. Working out must be very clear. Look at page 67 of your notes.

2. S is a subset of $\mathbb{R}^4$ which we know is a vector space. Hence we only need to check its not empty and the closure axioms.

   $\mathbf{0}$ is clearly in S therefore S is not empty.

   Let $\mathbf{x} = (x_1, x_2, x_3, x_4), \mathbf{y} = (y_1, y_2, y_3, y_4) \in S$. Then $(x_1 + y_1) - 3(x_2 + y_2) + 7(x_4 + y_4) = (x_1 - 3x_2 + 7x_4) + (y_1 - 3y_2 + 7y_4) = 0 + 0$ (since $\mathbf{x}, \mathbf{y}$ are in S), which equals 1. Hence $\mathbf{x} + \mathbf{y} \in S$ and so S is closed under (vector) addition.

   Let $\mathbf{x} = (x_1, x_2, x_3, x_4) \in S$ and $\lambda \in \mathbb{R}$. The $(\lambda x_1) - 3(\lambda x_2) + 7(\lambda x_4) = \lambda(x_1 - 3x_2 + 7x_4) = \lambda \times 0 = 0$. Thus S is closed under scalar multiplication.

   Therefore S is a vector space.

3. (a) The dimension of $M_{2,2}$ is 4. Thus 3 vectors (which in this case are matrices) can not span it.

   (b) Write the matrices as column vectors (in any way you like as long as you consistent) and stick as columns of a matrix. Row-reduce and find that you have a non-leading column. Therefore not linearly independent.

4. $T(\frac{3\pi}{4}, \frac{3\pi}{4}) \neq \frac{3\pi}{4} T(1, 1)$. Thus not linear.